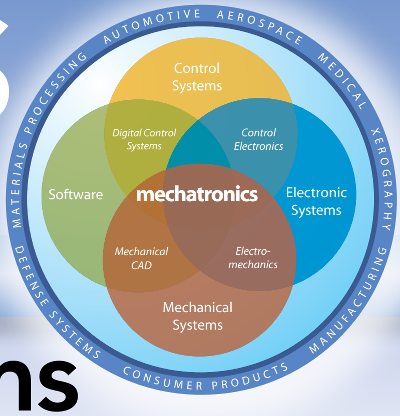
MECHATRONICS IN DESIGN

FRESH IDEAS ON INTEGRATING MECHANICAL SYSTEMS, ELECTRONICS, CONTROL SYSTEMS AND SOFTWARE IN DESIGN



Modeling: Block Diagrams and Transfer Functions

BY KEVIN CRAIG

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Modeling is the single most important activity in mechatronic system design and this article focuses on some techniques and tools engineers can use to create mathematical models from the various physical models of the physical systems they are investigating or designing.

Let's look at two situations as a basis of comparison. The first is the common situation where an engineer is designing a component or system and needs to predict its performance. The second, which is becoming more common in multidisciplinary systems in this global economy, occurs when subsystems are designed and manufactured by different vendors and must be brought together for final assembly.

The most common technique used for modeling linear, time-invariant systems is the block diagram, with the mathematical model represented as a transfer function. As an example of the first situation — designing your own component or system — let's use the simplest dynamic system for illustration, the first-order system. An example of this is the common electrical resistance-capacitance (RC) system. Shown below in Figure 1, is a first-order RC low-pass electrical filter. Once an engineer decides this physical model is a good representation of the actual physical system, the engineer can apply the appropriate Laws of Nature, here Kirchhoff's Voltage and Current Laws (KVL and KCL), to the physical model. Add to this the constitutive relations describing each model element voltage-current relationship (e = iR for a resistor and i = C de/dt for a capacitor) and you can generate a complete mathematical representation of the input-output behavior of the device.

At each port, input and output or the variables that define power — voltage and current — are identified, resulting in a complete description. The resulting mathematical model, consisting of differential equations, can be transformed to algebraic equations through the use of the Laplace transform or differential operator (D = d/dt). Once algebraic equations are obtained, ratios between input and output variables, the transfer functions, can be determined.

Now, to illustrate the second situation in which engineers need to connect two components to form a system, let's combine two identical RC circuits in series. An engineering student would most likely multiply two ideal transfer functions to get an overall input-output transfer function. This approach only works if steps have been taken to ensure the downstream RC circuit does not draw any power from the upstream RC circuit, by the insertion of a buffer op-amp in between, for example.

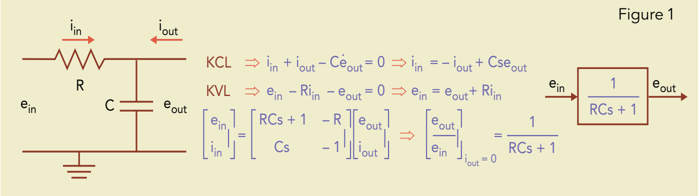

Figure 1

$$\text{KCL} \Rightarrow i_{in} + i_{out} - C\dot{e}_{out} = 0 \Rightarrow i_{in} = -i_{out} + Cse_{out}$$

$$\text{KVL} \Rightarrow e_{in} - Ri_{in} - e_{out} = 0 \Rightarrow e_{in} = e_{out} + Ri_{in}$$

$$\begin{bmatrix} e_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} RCs+1 & -R \\ Cs & -1 \end{bmatrix} \begin{bmatrix} e_{out} \\ i_{out} \end{bmatrix} \Rightarrow \left[\frac{e_{out}}{e_{in}}\right]_{i_{out}=0} = \frac{1}{RCs+1}$$

$$e_{in} \rightarrow \boxed{\frac{1}{RCs+1}} \rightarrow e_{out}$$

For this situation, there are three block-diagram/transfer-function methods an engineer can use to obtain the correct prediction. The most practical and relevant one is to use the concept of impedance. Impedance is defined at a port and is the ratio of effort to flow.

Figure 2, below right, shows the ratio of voltage to current at the input or output port of a component with some specified condition at the other port. If the output impedance of the upstream component Zo and the input impedance of the downstream component Zi are known either analytically or experimentally, then the overall transfer function for the interconnection of the components can be obtained.

Figure 2

$$\frac{e_{out}}{e_{in}} = G_{\substack{upstream\\ideal}} \left(\frac{1}{1+\dfrac{Z_0}{Z_i}} \right) G_{\substack{downstream\\ideal}}$$

Figure 3, below, shows the input impedance and output impedance for the RC circuit together with the ideal transfer function.

With this information, we can predict the response of the overall system when the two components are connected — in this case two identical RC circuits.

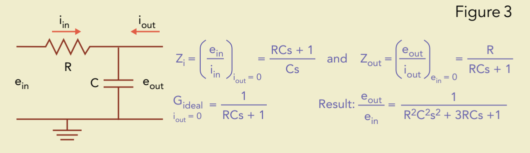

Figure 3

$$Z_i = \left(\frac{e_{in}}{i_{in}}\right)_{i_{out}=0} = \frac{RCs+1}{Cs} \quad \text{and} \quad Z_{out} = \left(\frac{e_{out}}{i_{out}}\right)_{e_{in}=0} = \frac{R}{RCs+1}$$

$$G_{\substack{ideal\\i_{out}=0}} = \frac{1}{RCs+1} \qquad \text{Result: } \frac{e_{out}}{e_{in}} = \frac{1}{R^2C^2s^2 + 3RCs + 1}$$

The importance of this approach is the impedances and ideal transfer functions can be measured experimentally for each component at separate locations. What's more, you can reliably predict system performance before the components are brought to the same location and connected.

Dive deeper into techniques to create mathematical models. Check out the MechatronicsZone:
http://rbi.ims.ca/5399-527