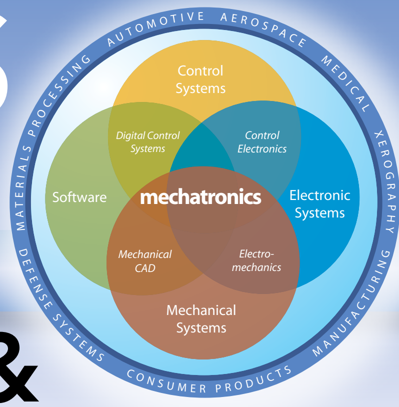
MECHATRONICS IN DESIGN

FRESH IDEAS ON INTEGRATING MECHANICAL SYSTEMS, ELECTRONICS, CONTROL SYSTEMS AND SOFTWARE IN DESIGN



Modeling: Linear Graphs & State-Equation Formulation

BY KEVIN CRAIG

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Our past columns have emphasized repeatedly that modeling is the single most important activity in mechatronics, which is becoming the design process of choice for successful multidisciplinary systems' engineers.

Today's practicing engineers need to be acquainted with the input-output, block-diagram, transfer-function dynamic system representation, which was discussed in our last article. And they also need to conversant with the energy-based, state-variable formulation. All these modeling tools are complementary and essential. And engineers can use various graphical representations of system structure, including block diagrams, linear graphs and bond graphs. Here, we discuss the linear graph, which stresses the concepts of continuity and compatibility.

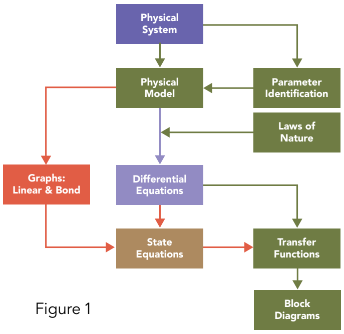


Figure 1

When modeling a dynamic system, a variable of interest may be one at the interface between the system and the environment, or it may be one internal to the system with no direct interaction with the environment. A state-variable mathematical representation of a dynamic system consists of a set of first-order, ordinary differential equations involving a complete and independent set of variables that are not unique. These are called state variables.

Engineers select as state variables quantities that are physically measurable and directly related to the energy stored in system elements. For example: velocity of a mass, current through an inductor, voltage across a capacitor and displacement of a spring. If engineers know the initial values of these state variables and the inputs to the system, they have all they need to determine system behavior. As shown in Figure 1, above, you can obtain these state equations by applying the laws of nature to the physical model or you can obtain them directly from a graphical representation of the physical model, in this case, a linear graph.

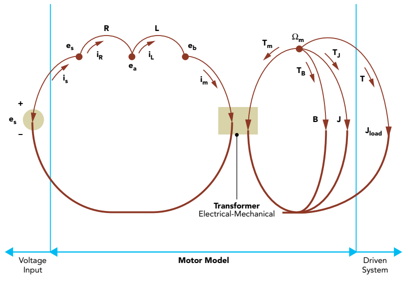
Depicting Multidisciplinary Structures

Engineers rely on the linear graph, which incorporates graphical line segments, to represent multidisciplinary system structure. This model is very similar in form to an electric circuit diagram. A node represents a point of interconnection of system elements and a line represents a system element. Some state variables are classified as "through variables," such as current and force, which measure transmission through an element. We refer to other state variables as "across variables." Examples would be voltage and velocity, which measure difference in state across an element.

The product of a "through variable" and an "across variable" is power. To obtain the state-variable equations, we apply a generalized version of Kirchhoff's current law, called a dynamic equilibrium relation, to describe the balance that must exist among the through variables. And we use a generalized version of Kirchhoff's voltage law, called a compatibility relation, to describe how the across variables are related. To these, you then must add the constitutive, purely empirical, physical relation for each element: $e = iR$ and $f = Bv$, relating the through variable to the across variable.

Let's apply these ideas to a brushed dc motor driving a load. The linear graph is shown in Figure 2, below, and the corresponding block diagram is shown in Figure 3, bottom.

Figure 2



If you want to learn more about linear graphs in modeling and analysis, consult these valuable references: "Introduction to System Dynamics" by L. Shearer, A. Murphy and H. Richardson, a classic work written in 1967; and a newer book (1997), "System Dynamics: An Introduction" by D. Rowell and D. Wormley. Both books should be on every engineer's shelf!

Our next article will discuss another important mechatronics' modeling tool, bond graphs. We'll follow this with a column comparing the three graphical representations — block diagrams, linear graphs and bond graphs — through a practical engineering example.

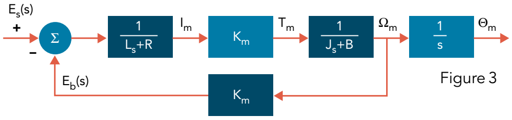


Figure 3

Learn more about energy-based, state-variable formulation at the Mechatronics Zone: http://rbi.ims.ca/5401-523.