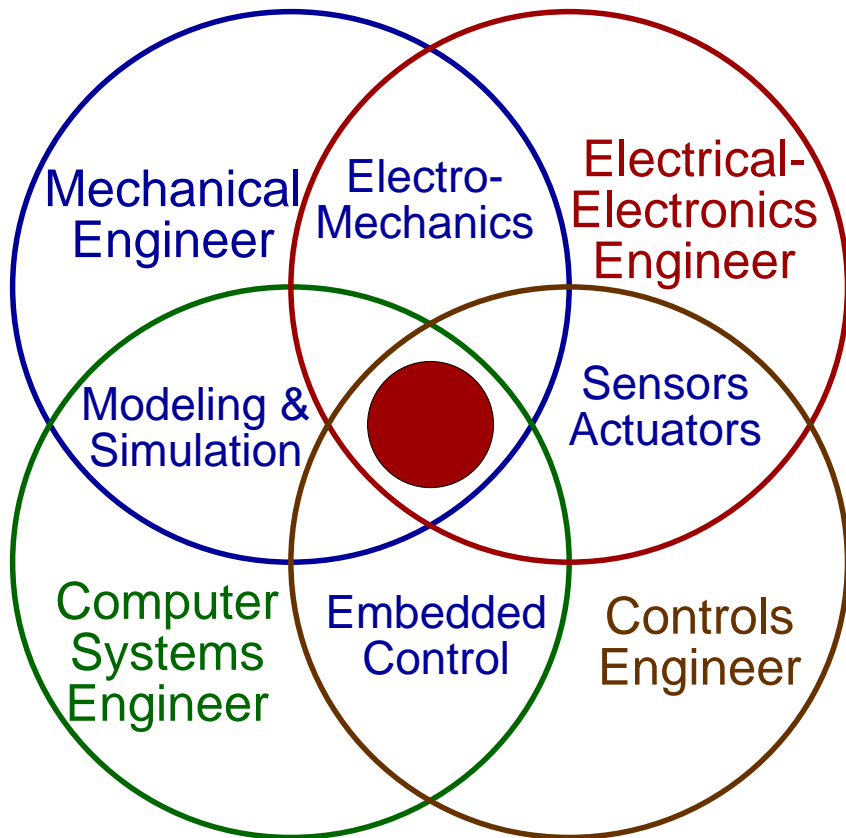
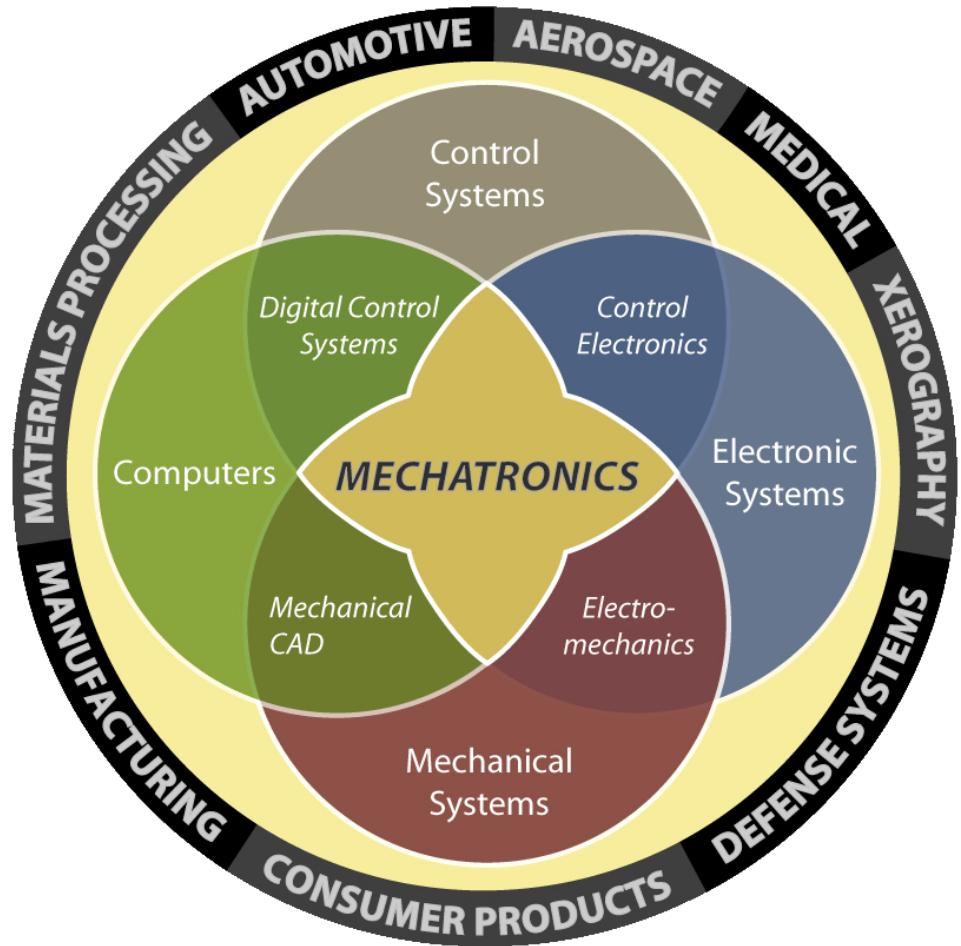


Acceleration Feedback



● Mechatronic System Design



- Introduction

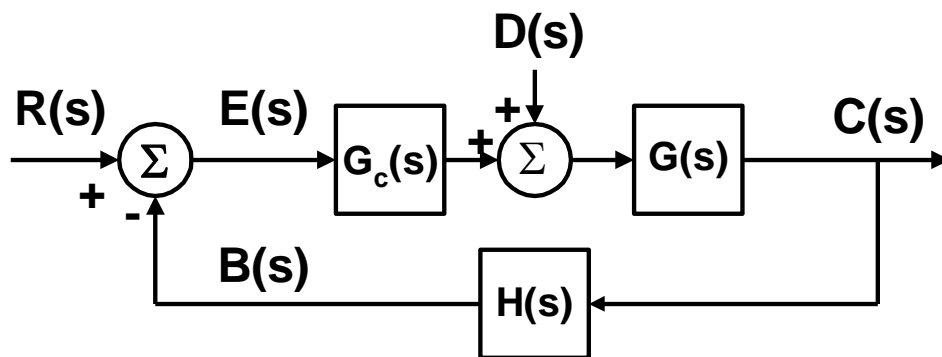
- The word inertia in everyday use suggests resistance to change and an unwillingness to act. This is hardly something we need in engineering practice to solve the urgent problems we all face. Even in a motion-system context, the idea of adding inertia to a system, i.e. adding mechanical mass, is not usually desirable as it slows down system response. One familiar exception is adding a flywheel to an engine or machine to smooth out speed fluctuations. Two of the most important benefits of feedback control are command following and disturbance rejection. Usually the focus of attention in a control system is on command following, but in many situations the ability of a system to reject disturbances, i.e., have high dynamic stiffness, is paramount.

- Disturbance Response

- Disturbance response for a control system is important, and in some applications, more important than command response.
- Disturbance response is more difficult to measure because disturbances are more difficult to produce than are commands.
- Both command response and disturbance response improve with high loop gains.
 - A high K_p provides a higher bandwidth and better ability to reject disturbances with high frequency content.
 - A high K_I helps the control system reject lower frequency disturbances.

- Setting K_I high has minimal effect on the command-response Bode plots. K_I is aimed at improving response to disturbances, not commands. In fact, the process of tuning K_I is essentially to raise it as high as possible without significant impact on the command response. K_I is not noticeable in the command response until it is high enough to cause peaking and instability. High K_I provides unmistakable benefit in disturbance response.
- In addition, **Disturbance-Compensated Feedforward Control** (see slides 32-34) aids disturbance response by using measured or estimated disturbances to improve disturbance response.
- Sometimes disturbance response is referred to by its inverse – **disturbance rejection or dynamic stiffness**. Control systems need to have **high dynamic stiffness** (disturbance rejection) and low disturbance response.

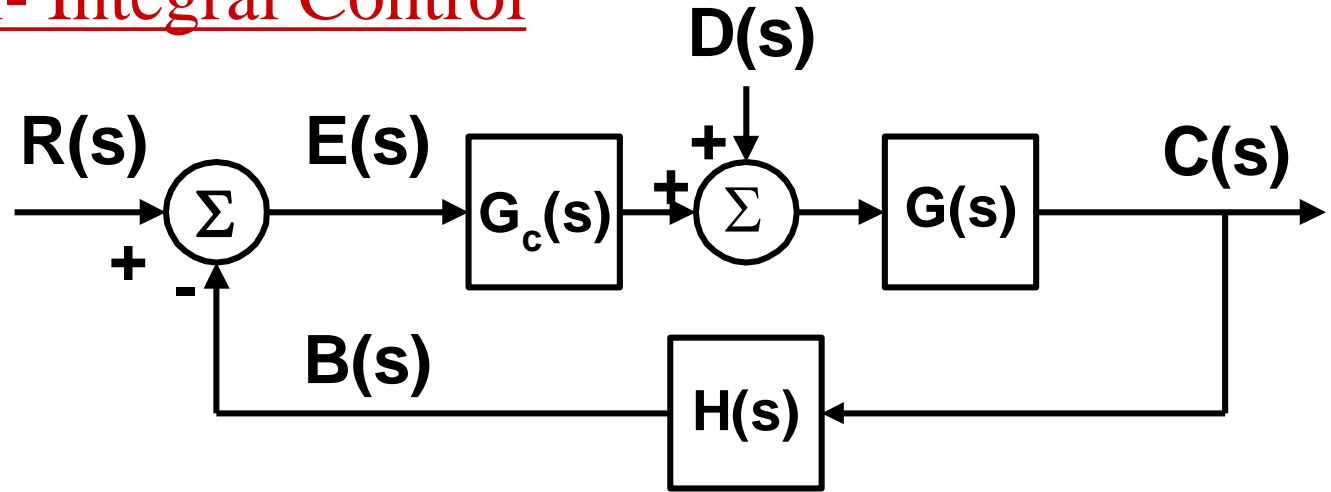
- Dynamic Stiffness is a measure of how much force is required to move a system as opposed to disturbance response, which is a measure of how much the system moves in the presence of a force. A system that is very stiff responds little to disturbances.
- The control system cannot reject the disturbance perfectly because the disturbance is detected only after it moves the output; the controller cannot react until system output has been disturbed.
- Disturbance response is defined as the response of the system output $C(s)$ to the disturbance $D(s)$.



$$\frac{C(s)}{D(s)} = \frac{G(s)}{1 + G_c(s)G(s)H(s)}$$

- One way to improve disturbance response is to use slow-moving plants, e.g., large inertia, high capacitance, to provide low plant gains. Reduce $G(s)$. This is a time-proven technique. Large flywheels smooth motion; large inductors and capacitors smooth voltage output.
- A second way to improve disturbance response is to increase the gains of the controller, $G_C(s)$. This is how integral gains grant systems perfect response to DC inputs: the gain of the ideal integrator at 0 Hz is infinite, driving up the magnitude of the transfer function denominator and, thus, driving down the disturbance response.
- At other frequencies, unbounded gain is impractical, so AC disturbance response is improved with high gains but not cured entirely.

Proportional- Integral Control



$$G(s) = \frac{1}{Js}$$

$$G_C(s) = K_P + \frac{K_I}{s}$$

$$H(s) = 1$$

$$\frac{C(s)}{D(s)} = \frac{G(s)}{1 + G_C(s)G(s)H(s)}$$

$$= \frac{s}{Js^2 + K_P s + K_I}$$

High-Frequency Range

Low-Frequency Range

Mid-Frequency Range

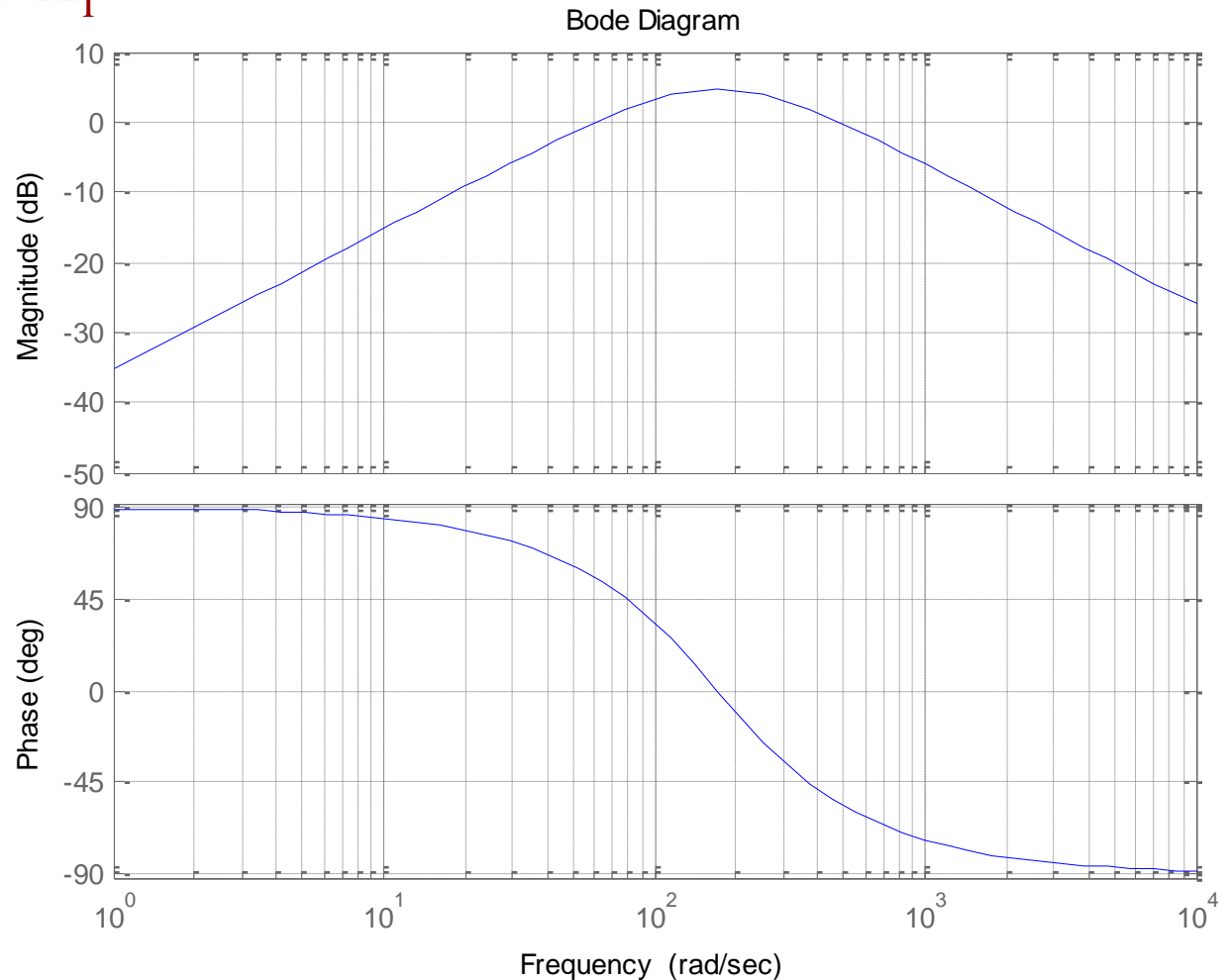
$$\frac{C}{D}(s) = \frac{s}{Js^2 + K_p s + K_I}$$

Disturbance Response

$$J = 0.002$$

$$K_p = 0.58$$

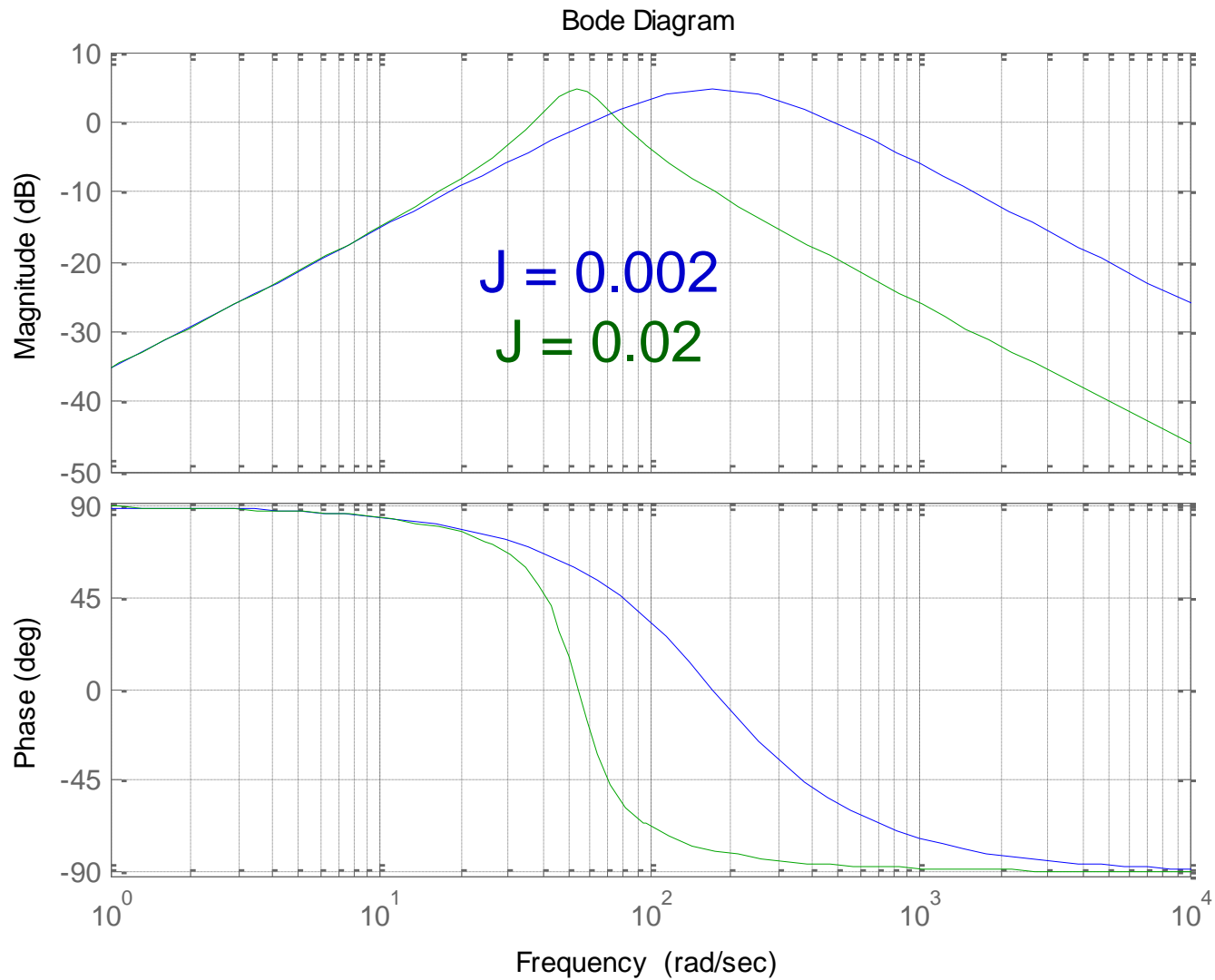
$$K_I = 58$$



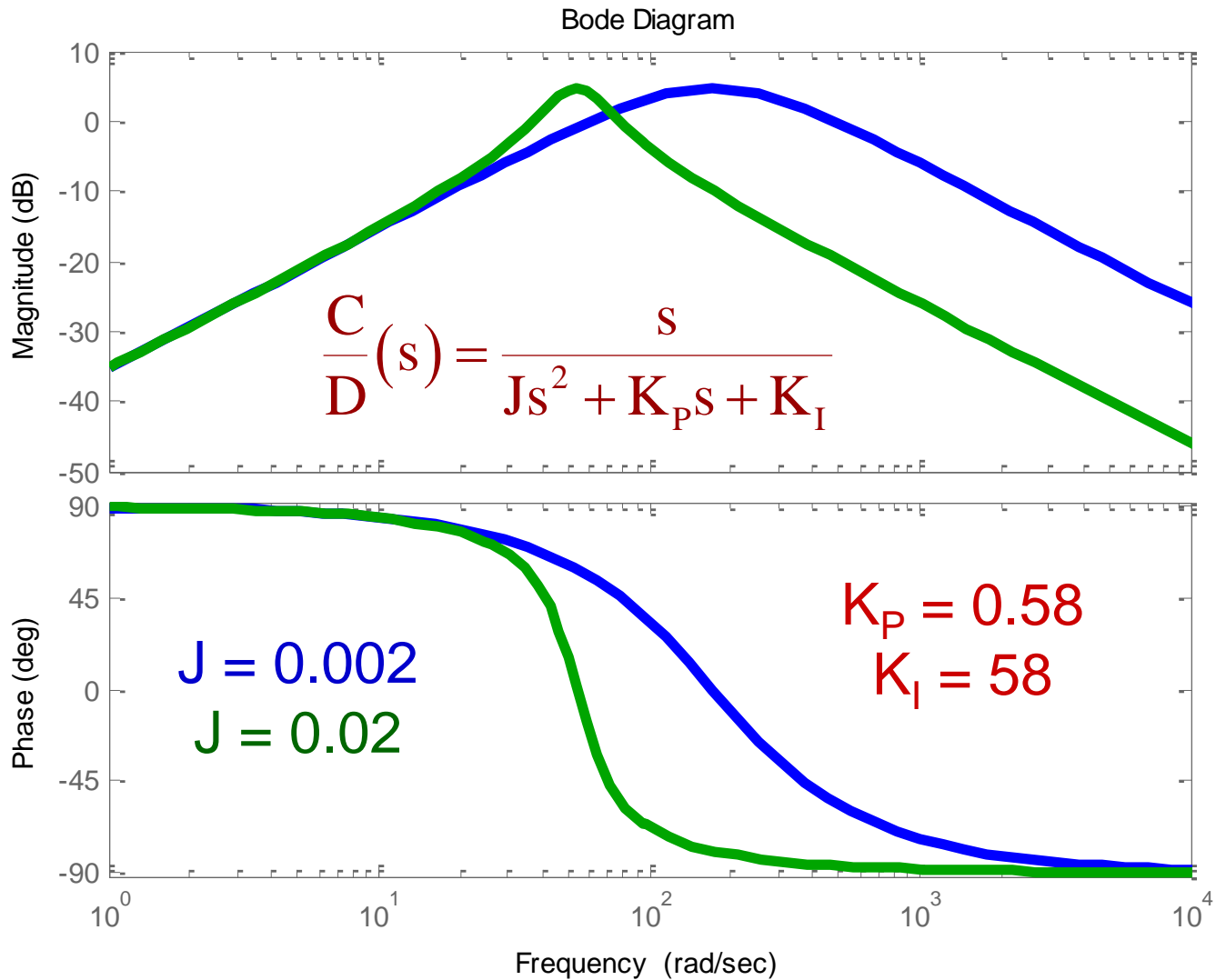
Increase J
 Increase K_p
 Increase K_I

What Happens?

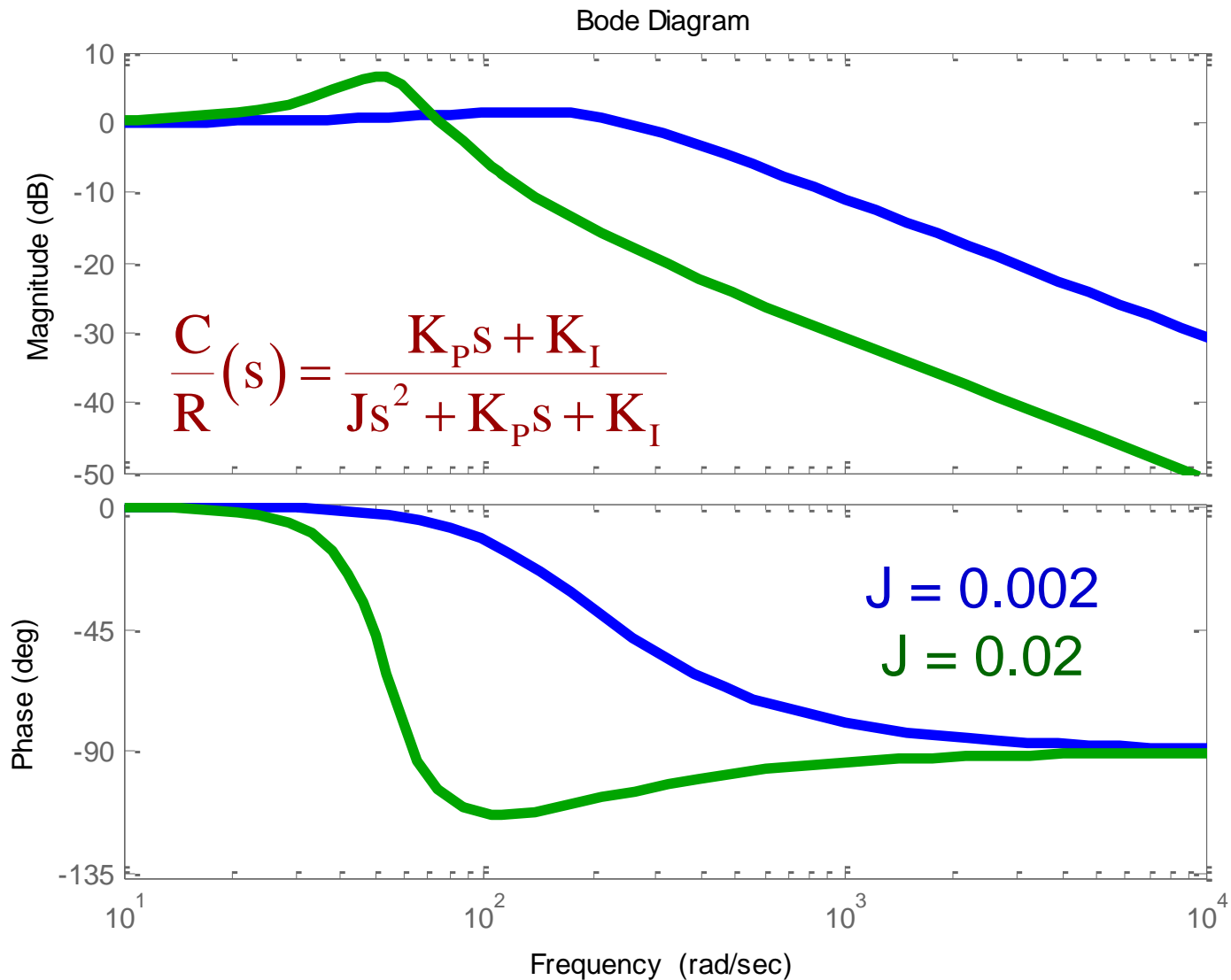
$$\frac{C}{D}(s) = \frac{s}{Js^2 + K_p s + K_I}$$



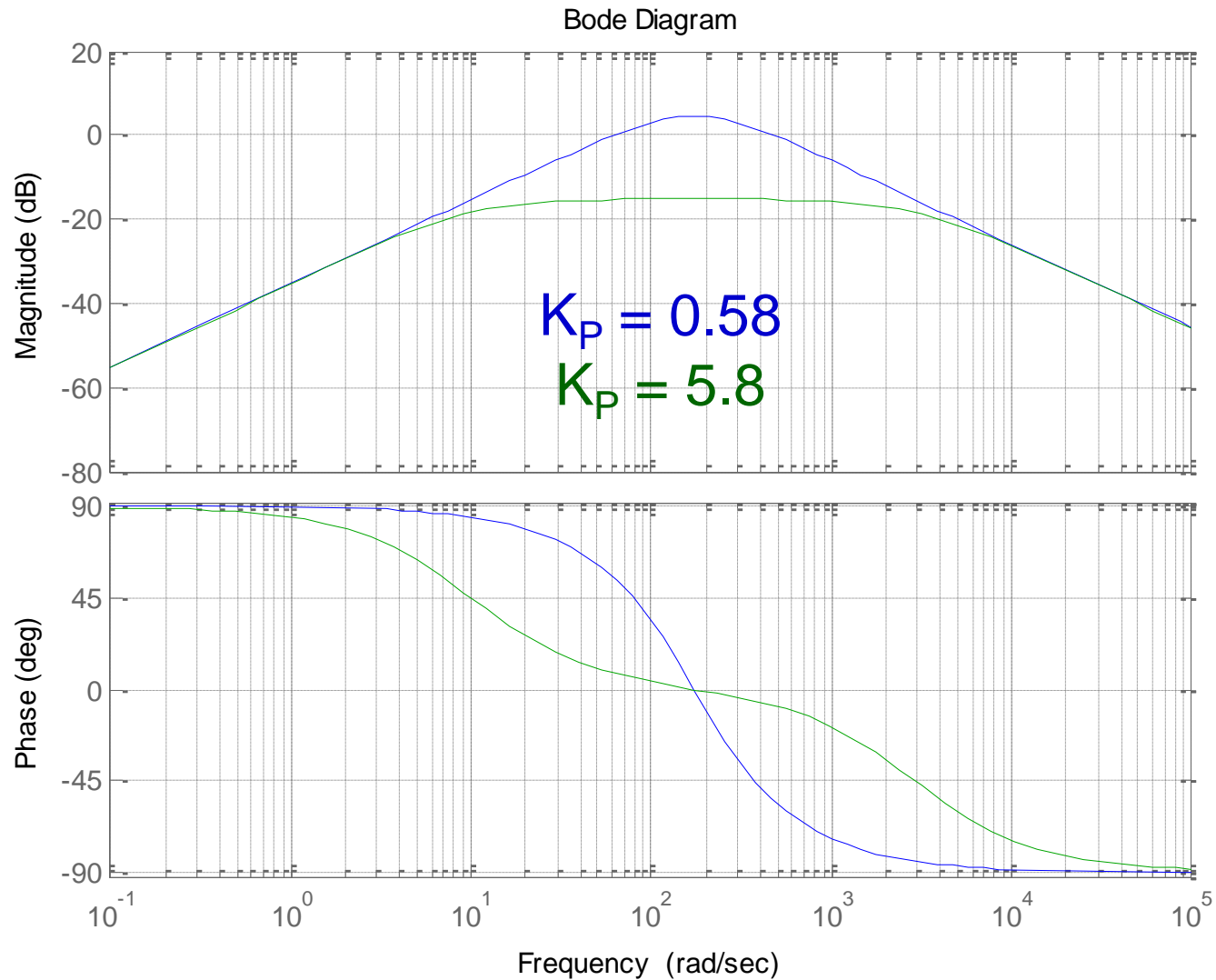
Disturbance Response



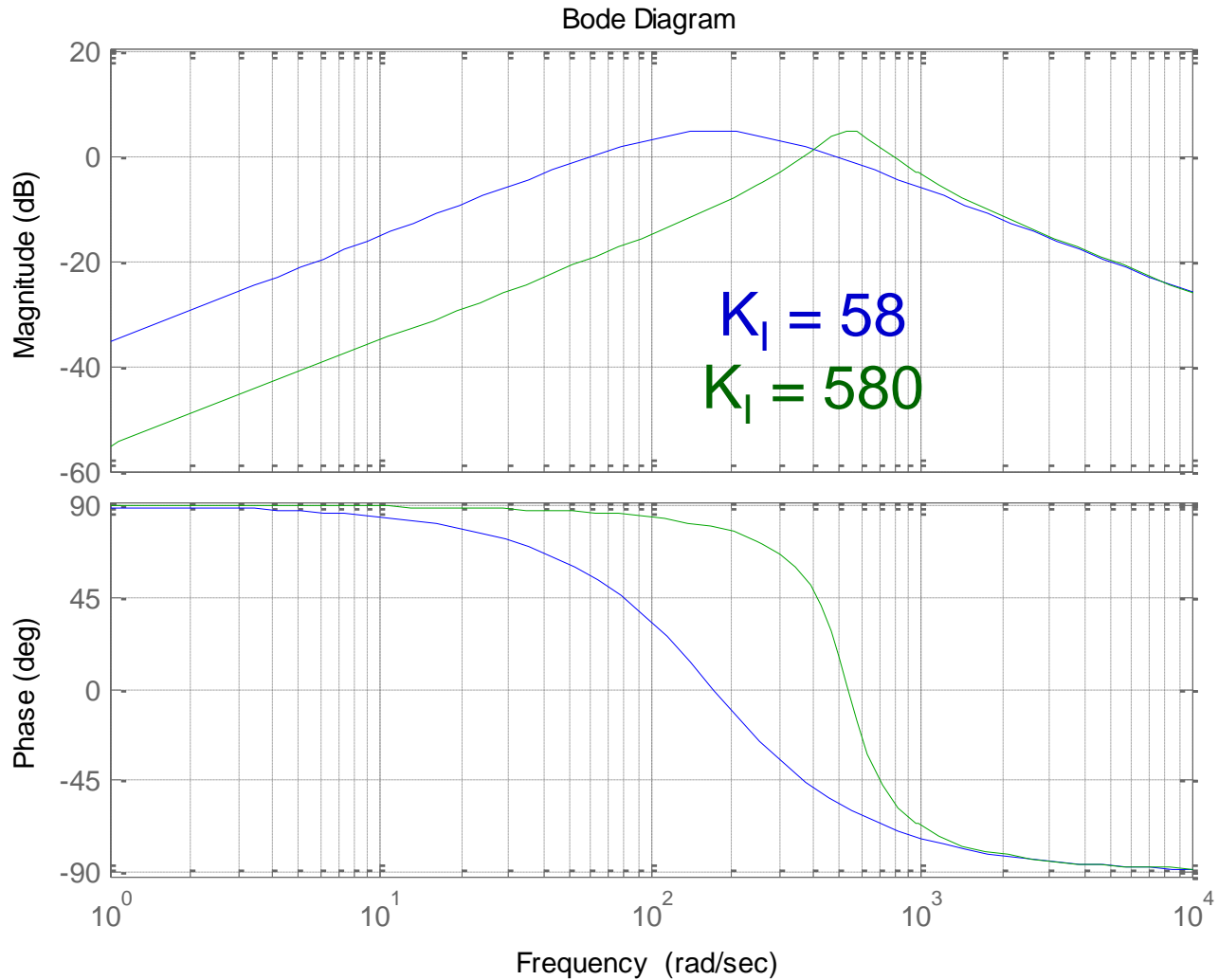
Command Response



$$\frac{C}{D}(s) = \frac{s}{Js^2 + K_p s + K_I}$$



$$\frac{C}{D}(s) = \frac{s}{Js^2 + K_p s + K_I}$$



- Some Observations

- Increasing the value of J reduces (improves) the disturbance response in the higher frequencies. Disturbance response from the inertia improves as frequency increases ($1/J_s$ term).
- However, increasing J degrades the closed-loop command-following response of the system.
- In the medium frequency range the $1/K_p$ term dominates. A larger proportional gain helps in the medium frequencies.
- In the lowest frequency range, the s/K_I term dominates. Larger integral gain improves the low-frequency disturbance response.

- Electronic Inertia
- Inertia Adaptation
- Acceleration Feedback
- What is Inertia?

What do these mean ?
Why is this important ?

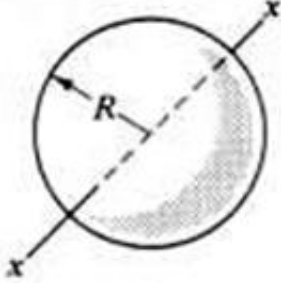
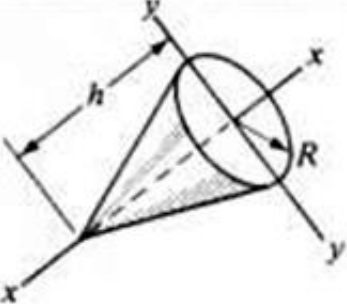
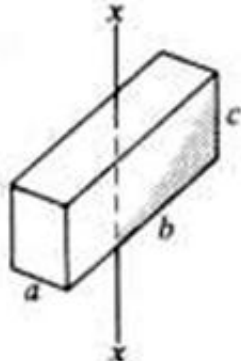
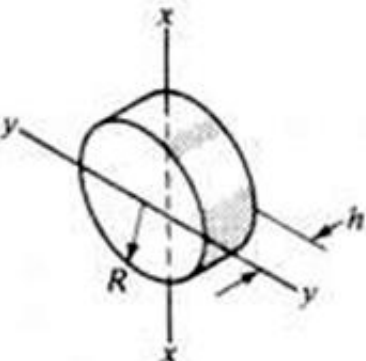
- Mass

- Mass is the quantitative measure of the inertia or resistance to change in motion of a rigid body.

- Mass Moment of Inertia

- The mass moment of inertia of a rigid body is a measure of the radial distribution of the body's mass with respect to an axis through some point. It represents the body's resistance to change in angular motion about the axis through the point.

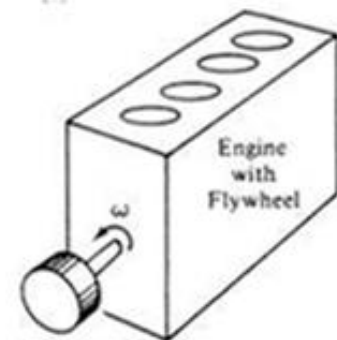
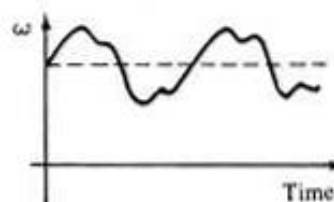
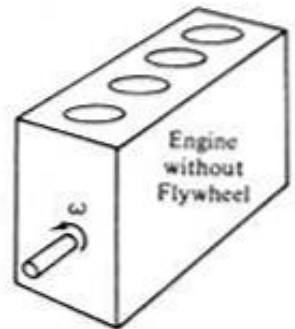
Mass Moments of Inertia for Some Common Shapes

 <p>Sphere</p> $J_{xx} = \frac{2MR^2}{5}$	 <p>Cone</p> $J_{xx} = \frac{3MR^2}{10}$ $J_{yy} = \frac{M(3R^2 + 2h^2)}{20}$
 $J_{xx} = \frac{M(a^2 + b^2)}{12}$	 $J_{xx} = M \left(\frac{h^2}{12} + \frac{R^2}{4} \right)$ $J_{yy} = \frac{M}{12} (4h^2 + 3R^2)$
<p>All xx Axes Go through the Mass Center</p>	

- Is Inertia Good or Bad?

- A designer rarely inserts a component for the purpose of adding inertia; the mass or inertia element often represents an undesirable effect which is unavoidable since all materials have mass.
- There are some applications in which mass itself serves a useful function, e.g., flywheels.

Flywheels are used as energy-storage devices or as a means of smoothing out speed fluctuations in engines or other machines.



Inertia Element

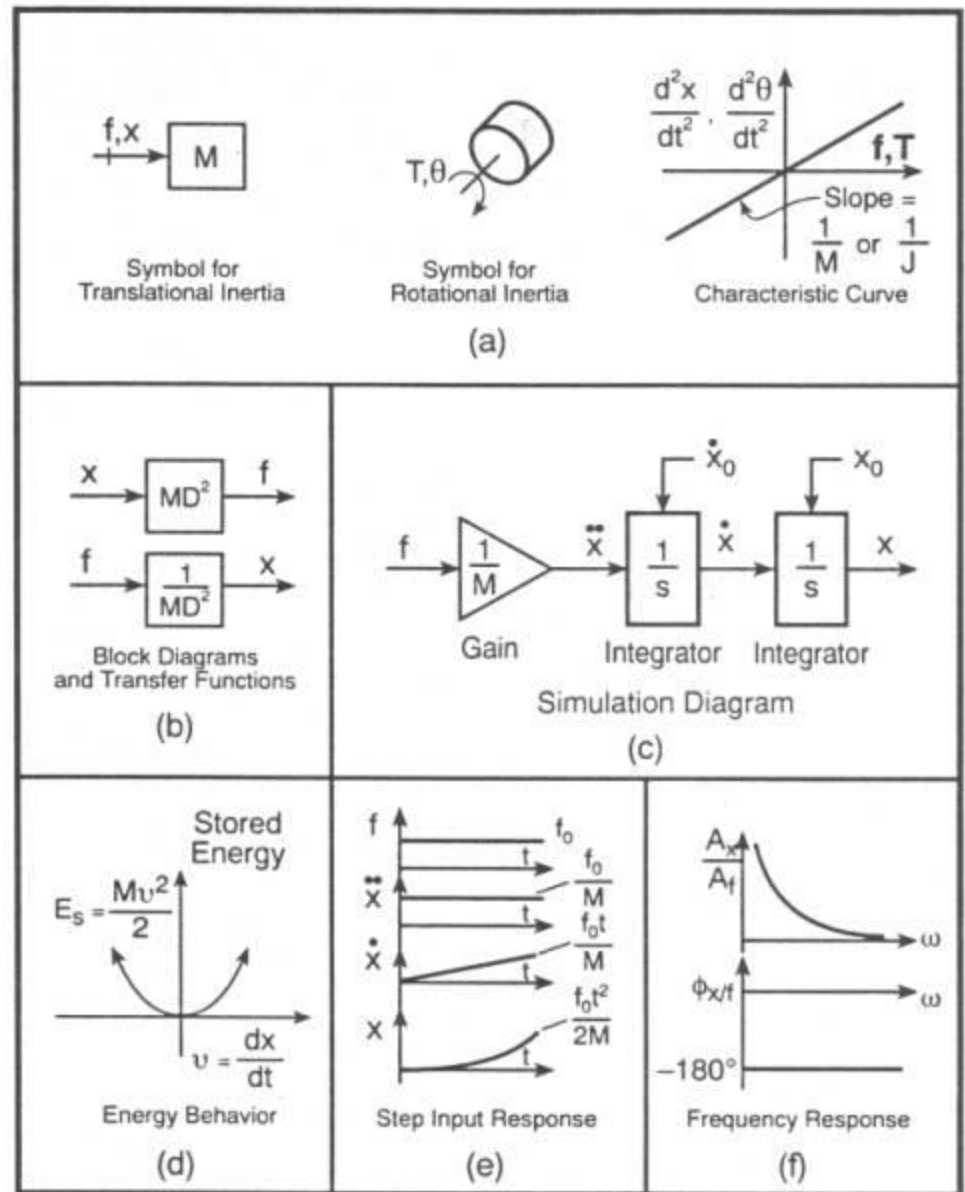
Real inertias may be impure (have some springiness and friction) but are very close to ideal.

$$M\ddot{x} = f \qquad J\ddot{\theta} = T$$

$$\frac{x}{f}(D) = \frac{1}{MD^2} \qquad \frac{\theta}{T}(D) = \frac{1}{JD^2}$$

Inertia Element stores energy as kinetic energy:

$$\frac{Mv^2}{2} \quad \text{or} \quad \frac{J\omega^2}{2}$$



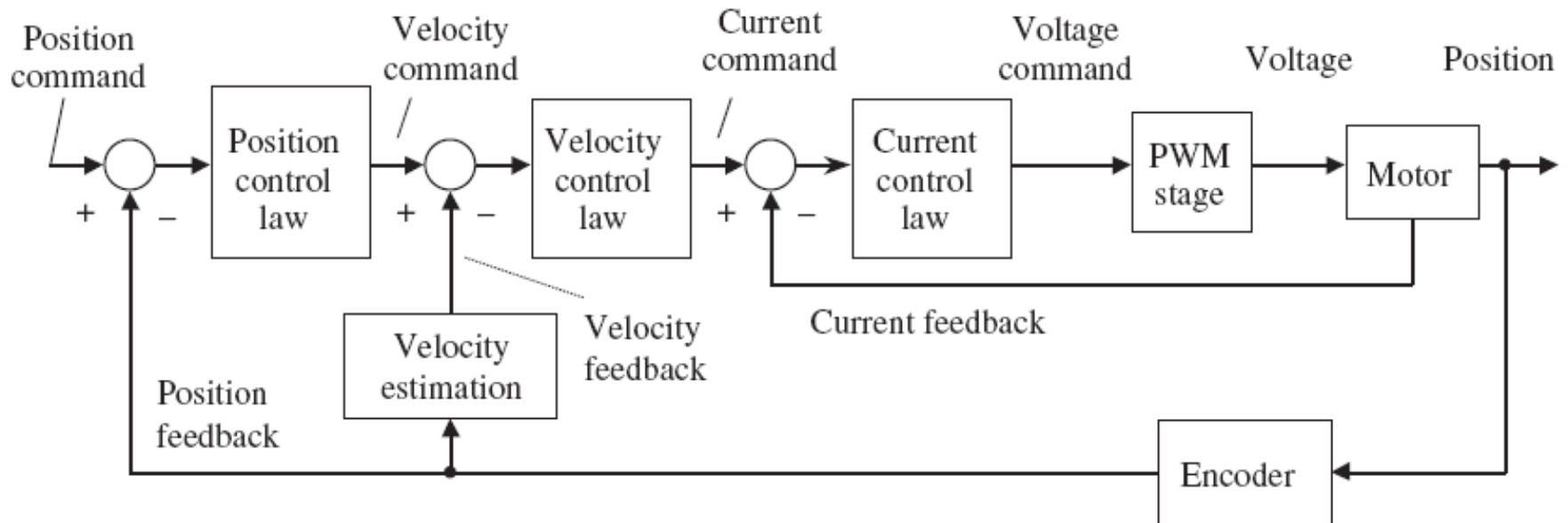
- A step input force applied to a mass initially at rest causes an instantaneous jump in acceleration, a ramp change in velocity, and a parabolic change in position.
- The frequency response of the inertia element is obtained from the sinusoidal transfer function:

$$\frac{x}{f}(i\omega) = \frac{1}{M(i\omega)^2} = \frac{1}{M\omega^2} \angle -180^\circ$$

- At high frequency, the inertia element becomes very difficult to move.
- The phase angle shows that the displacement is in a direction opposite to the applied force.

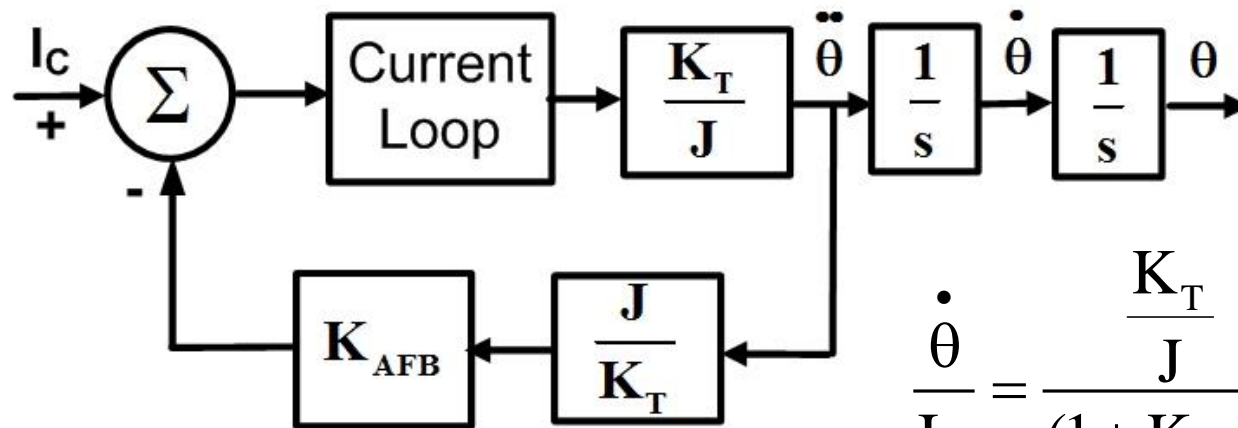
- So What is Acceleration Feedback?

- We know about current, velocity, and position feedback, as shown below. Cascaded control loops are very common in industrial motion control.



- So what is acceleration feedback? How does one obtain an acceleration feedback signal? It has been shown to improve system performance, but how?

- Acceleration feedback works by slowing the motor in response to measured or estimated acceleration. The acceleration of the motor is measured, scaled by the ratio of inertia J to motor torque constant K_T , then by a gain K_{AFB} , and then used to reduce the acceleration (current command).
- The larger the actual acceleration, the more the current command is reduced.
- K_{AFB} has a similar effect to increasing inertia; hence the alternate name electronic inertia / flywheel.



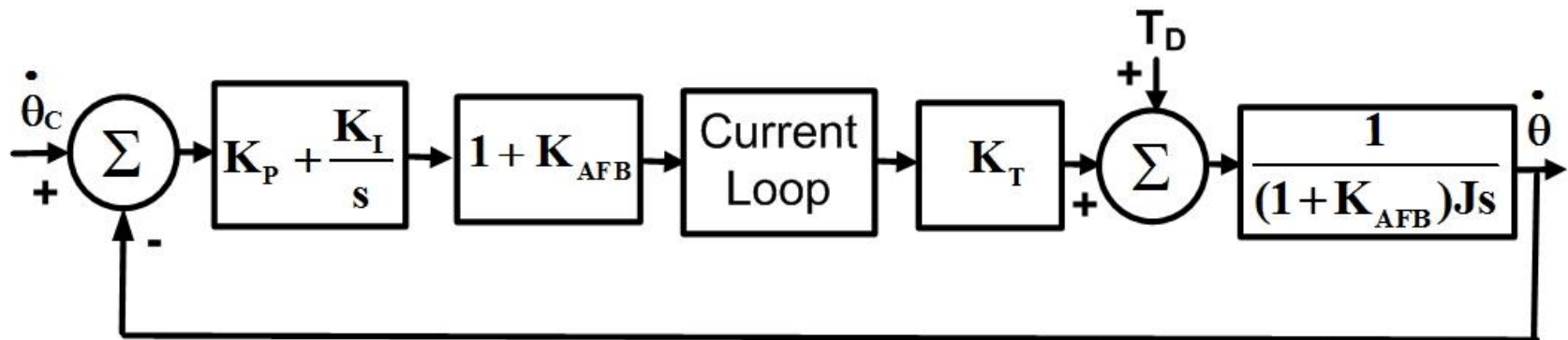
$$\frac{\dot{\theta}}{I_C} = \frac{\frac{K_T}{J}}{(1 + K_{AFB})} \frac{1}{s} = \frac{K_T}{(1 + K_{AFB})Js}$$

- The effect of acceleration feedback is easily seen by calculating the transfer function of the previous diagram.
 - Assume that the current loop dynamics are ideal.
 - The resulting transfer function is:

$$\frac{\dot{\theta}}{I_C} = \frac{\frac{K_T}{J}}{(1 + K_{AFB})} \frac{1}{s} = \frac{K_T}{(1 + K_{AFB})Js}$$

- It is clear that any value of $K_{AFB} > 0$ will have the same effect as increasing the total inertia, J , by the factor $(1 + K_{AFB})$.
- Hence, K_{AFB} can be thought of as electronic inertia.

- The primary effect of feeding back acceleration is to increase the effective inertia. However, this alone produces little benefit. The increase in effective inertia actually reduces loop gain, reducing system response rates.
- The benefits of acceleration feedback are realized when control-loop gains are scaled up by the amount that the inertia increases, that is, by the factor $(1 + K_{AFB})$, as shown below.



- In the diagram, as K_{AFB} increases, the effective inertia increases, and the loop gain is fixed so that the stability margins and command response are unchanged.
- The closed-loop transfer function is:

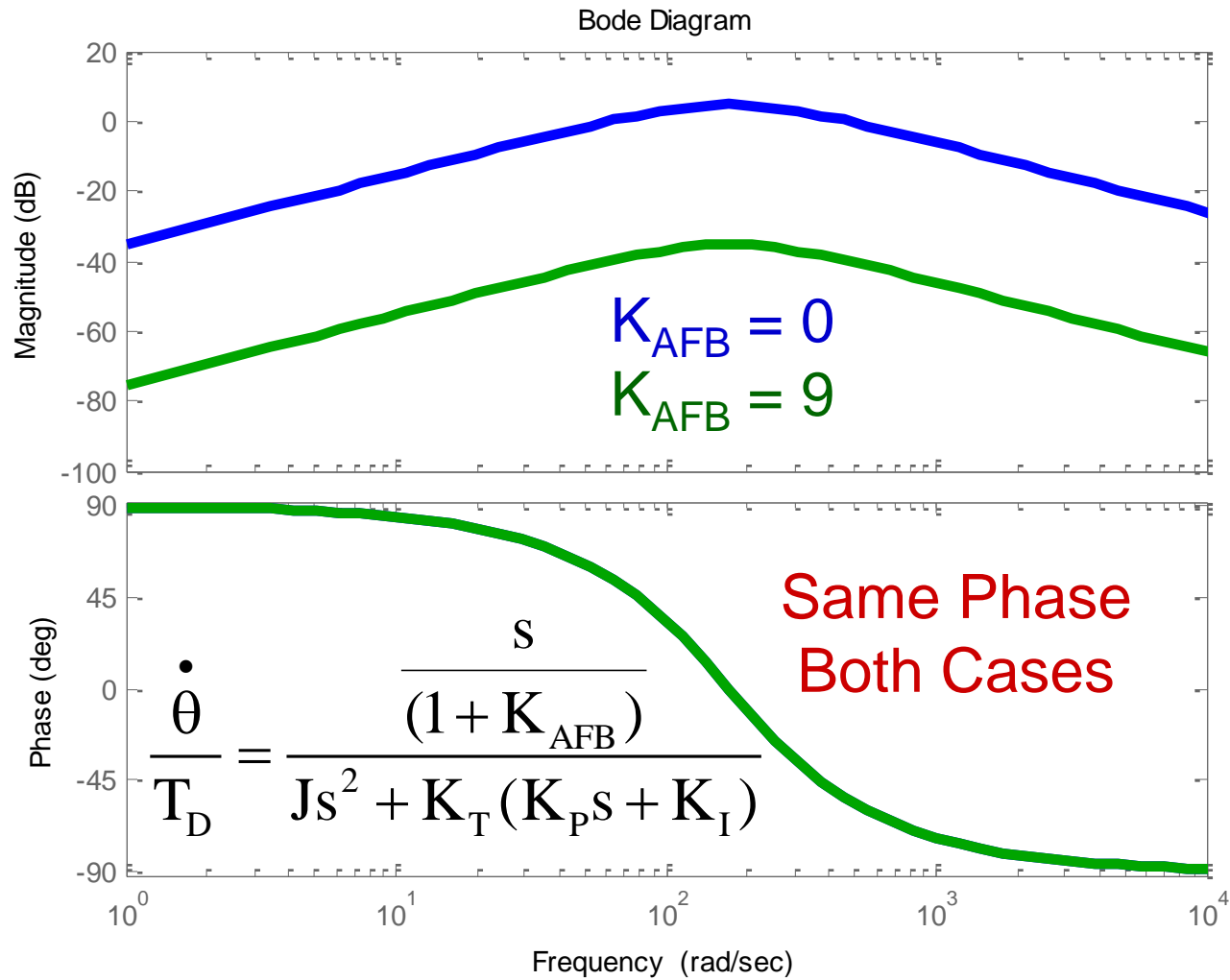
$$\frac{\dot{\theta}}{\dot{\theta}_C} = \frac{K_T (K_P s + K_I)}{J s^2 + K_T (K_P s + K_I)}$$

- Notice that the command response is unaffected by the value of K_{AFB} . This is because the loop gain increases in proportion to the inertia, producing no net effect.

- The disturbance response, unlike the command response is improved by acceleration feedback, as shown.

$$\frac{\dot{\theta}}{T_D} = \frac{\frac{s}{(1 + K_{AFB})}}{Js^2 + K_T(K_P s + K_I)}$$

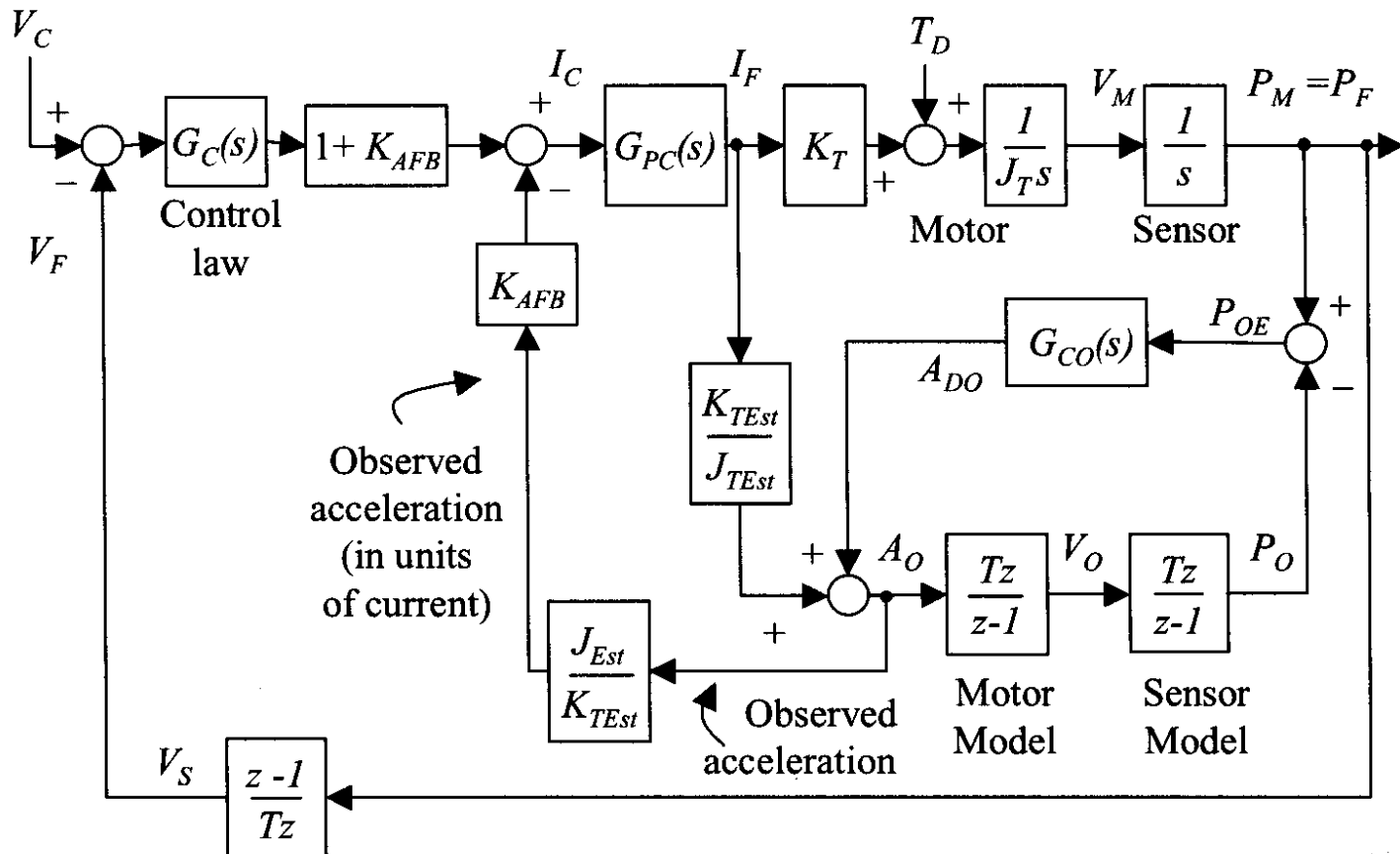
- For this idealized case, the disturbance response is improved through the entire frequency range in proportion to the term $(1 + K_{AFB})$. Unfortunately, such a result is impractical. The improvement cannot be realized significantly above the bandwidth of the power converter (current loop). This is clear upon inspection as the acceleration feedback signal cannot improve the system at frequencies where the current loop cannot inject current.



- The second limitation on acceleration feedback is the difficulty in measuring acceleration. While there are acceleration sensors that are used in industry, few applications can afford either the increase in cost or the reduction in reliability brought by an additional sensor and its associated wiring.
- Also, designers generally prefer to use only one feedback transducer, e.g., optical encoder or electromagnetic resolver. So the acceleration signal must be derived from the position signal. But differentiation produces noisy signals! And passing this signal through filters adds phase delay.

- One solution to the problem is to use observed acceleration rather than measured. Of course, the acceleration can be observed only within the capabilities of the observer configuration; this limits the frequency range over which the ideal results can be realized.
- Observed acceleration is a suitable alternative for acceleration feedback in many systems where using a separate acceleration sensor is impractical.

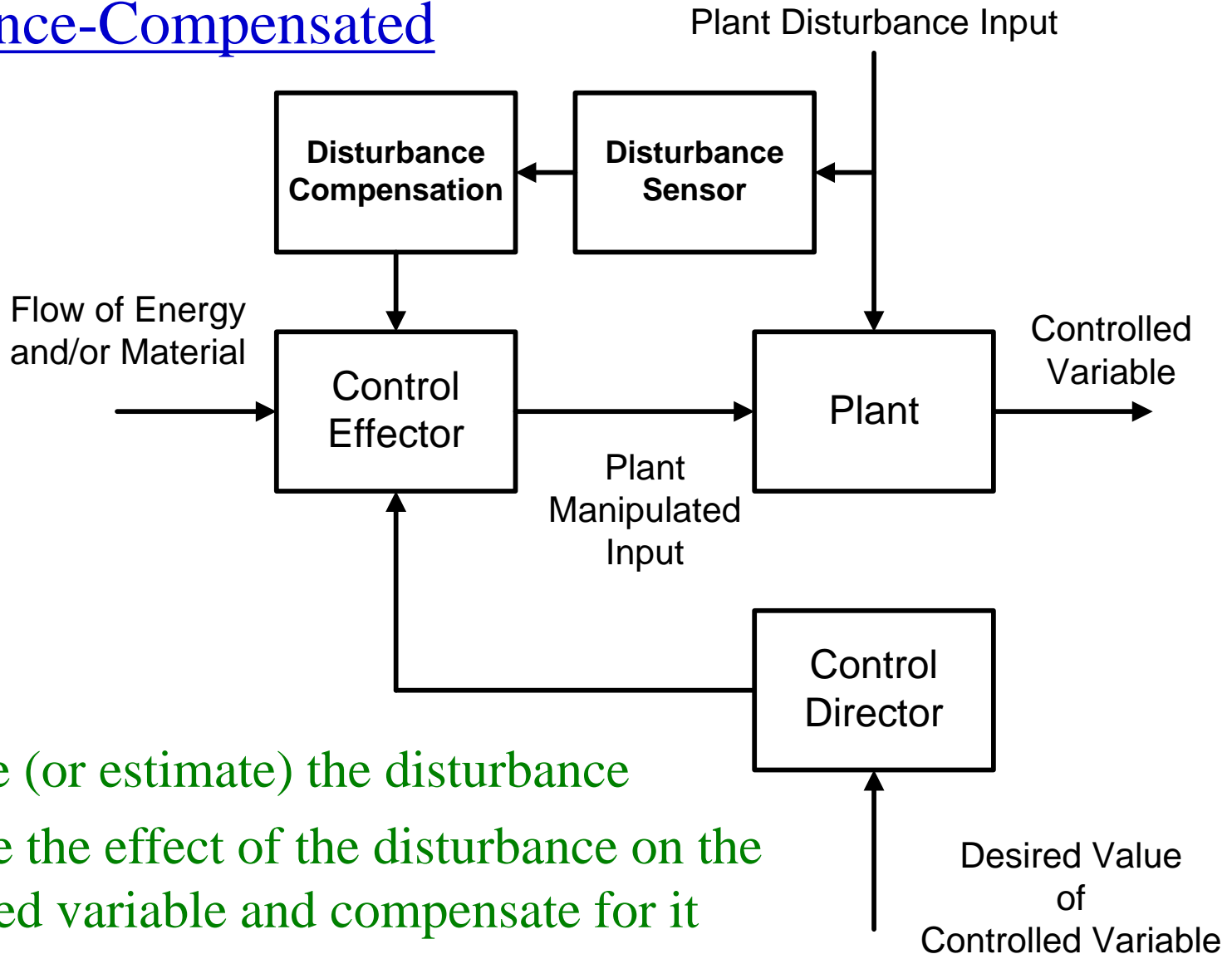
- In the diagram below, the observed acceleration A_O is scaled to current units and deducted from the current command. The term $(1 + K_{AFB})$ scales the control-law output, as before.



- Summary

- The effect of acceleration feedback is to introduce the same resistance to disturbance as pure inertia would, but the increase in dynamic stiffness occurs across all frequencies, not just at high frequency. The command closed-loop transfer function, and hence the closed-loop bandwidth, is unchanged as the feedback gains are adjusted.
- The current feedback loop also has the effect of increasing dynamic stiffness, but it operates before the disturbance input, while the acceleration feedback loop acts after the disturbance input. The controller can respond faster because the acceleration feedback signal contains the disturbance input.
- A robust acceleration feedback signal is required. This can be accomplished through differentiation of a position sensor signal and filtering or through the use of an observer.

Open-Loop Input-Compensated Feedforward Control: Disturbance-Compensated



- Measure (or estimate) the disturbance
- Estimate the effect of the disturbance on the controlled variable and compensate for it

- Disturbance-Compensated Feedforward Control
 - Basic Idea: Measure an input disturbance to the plant and take corrective action (adjust the manipulated variable) before it upsets the process (causes the controlled variable to deviate from its set point). This measurement provides an early warning that the controlled variable will be upset some time in the future.
 - In contrast, a feedback controller does not take corrective action until after the disturbance has upset the process and generated an error signal.
 - **This controller does not use feedback!** However, it is usually combined with feedback control so that the important features of feedback are retained in the overall strategy.

- There are several disadvantages to disturbance-compensated feedforward control:
 - The disturbance must be measured on line. In many applications, this is not feasible.
 - The quality of the feedforward control depends on the accuracy of the process model; one needs to know how the controlled variable responds to changes in both the disturbance and manipulated variables.
 - Ideal feedforward controllers that are theoretically capable of achieving perfect control may not be physically realizable. Fortunately, practical approximations of these ideal controllers often provide very effective control.