

Analogies

- Analogies Give Engineers Insight!
 - *Insight based on fundamentals is the key to innovative multidisciplinary problem solving.*
 - A person trying to explain a difficult concept will often say “Well, the analogy is ...” The use of analogies in everyday life aids in understanding and makes everyone better communicators. Mechatronic systems depend on the interactions among mechanical, electrical, magnetic, fluid, thermal, and chemical elements, and most likely combinations of these. They are truly multidisciplinary and the designers of mechatronic systems are from diverse backgrounds. Knowledge of physical system analogies can give design teams a significant competitive advantage.

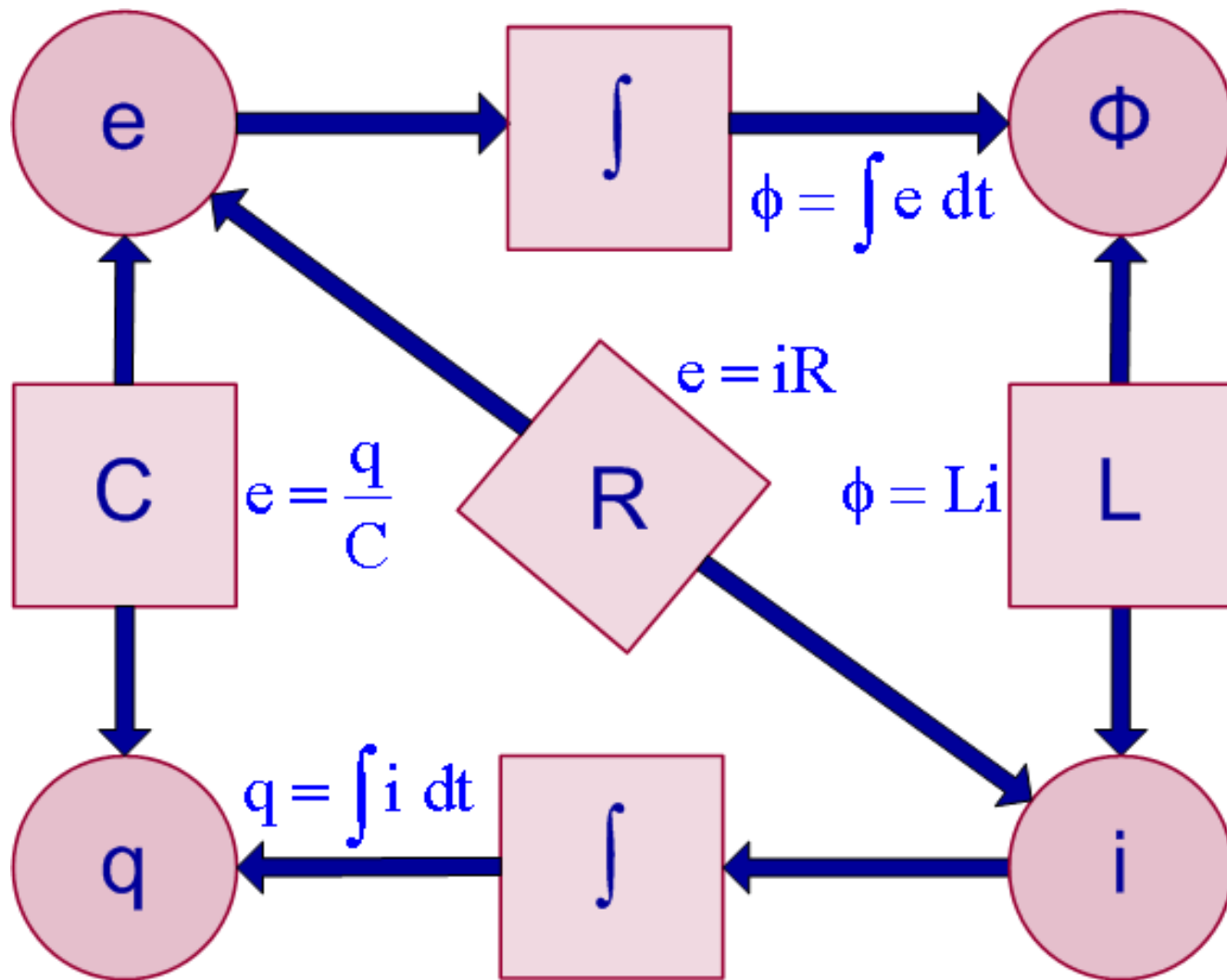
Electrical – Mechanical Analogies

- A signal, element, or system which exhibits mathematical behavior identical to that of another, but physically different, signal, element, or system is called an analogous quantity or analog.
- Let's explore the common electrical-mechanical analogy.
 - These systems are modeled using combinations of pure (only have the characteristic for which they are named) and ideal (linear in behavior) elements: resistor (R), capacitor (C), and inductor (L) for electrical systems and damper (B), spring (K), and mass (M) for mechanical systems. The variables of interest are voltage (e) and current (i) for electrical systems and force (f) and velocity (v) for mechanical systems.

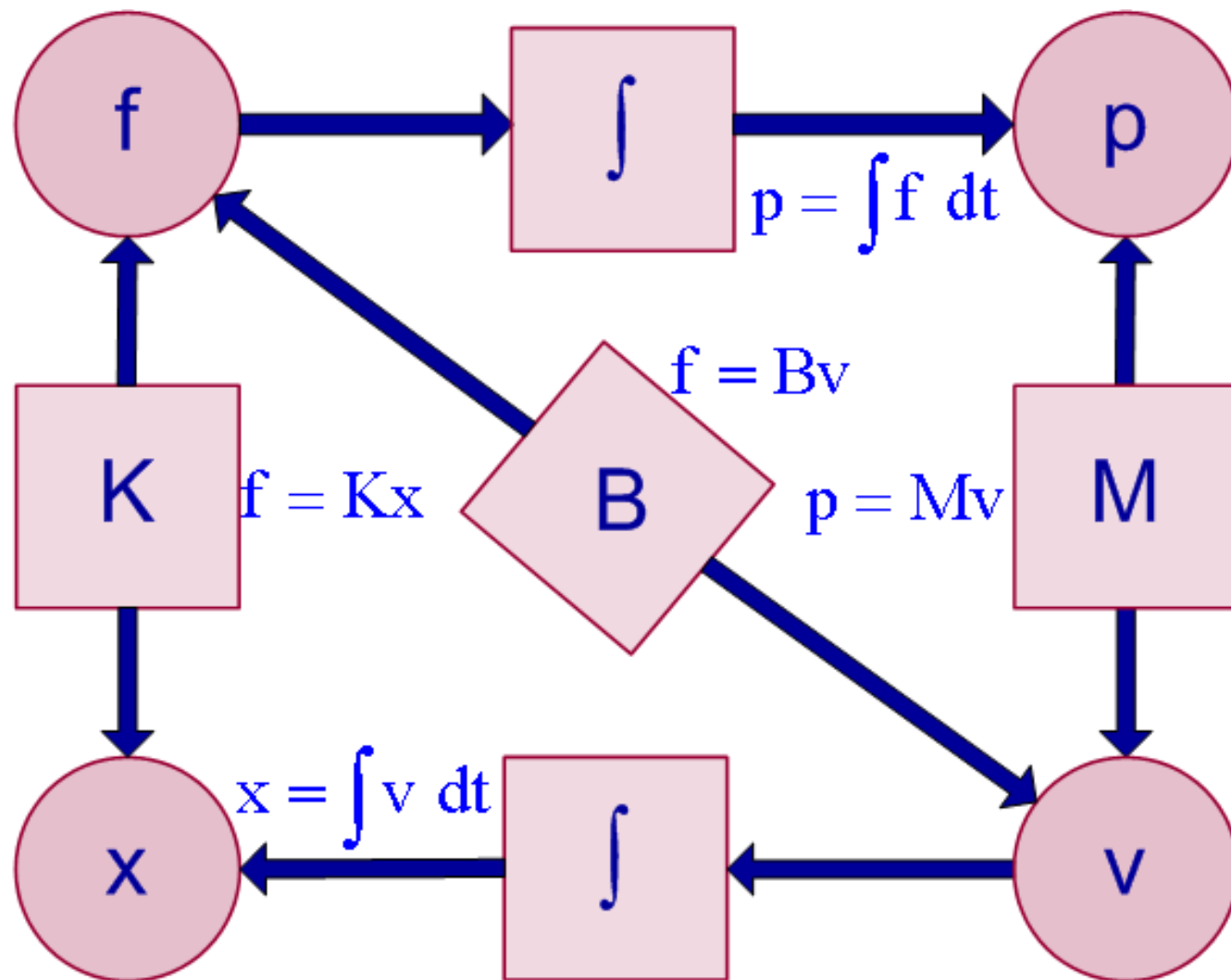
- Force causes velocity, just as voltage causes current.
- A damper dissipates mechanical energy into heat, just as a resistor dissipates electrical energy into heat.
- Springs and masses store energy in two different ways (potential energy and kinetic energy), just as capacitors and inductors store energy in two different ways (electric field and magnetic field).

Spring Potential Energy	$\frac{1}{2} Kx^2 = \frac{1}{2} \frac{(Kx)^2}{K} = \frac{1}{2} \frac{f^2}{K}$	\Leftrightarrow	$\frac{1}{2} Ce^2 = \frac{1}{2} \frac{q^2}{C}$	Capacitor Electric Field Energy
Mass Kinetic Energy	$\frac{1}{2} Mv^2$	\Leftrightarrow	$\frac{1}{2} Li^2$	Inductor Magnetic Field Energy

- The product $(f)(v)$ represents instantaneous mechanical power; $(e)(i)$ represents instantaneous electrical power.



Model Structure for Electrical Systems



Model Structure for Mechanical Systems

force $f \Leftrightarrow$ voltage e
 velocity $v \Leftrightarrow$ current i
 damper $B \Leftrightarrow$ resistor R
 spring $K \Leftrightarrow$ capacitor $1/C$
 mass $M \Leftrightarrow$ inductor L

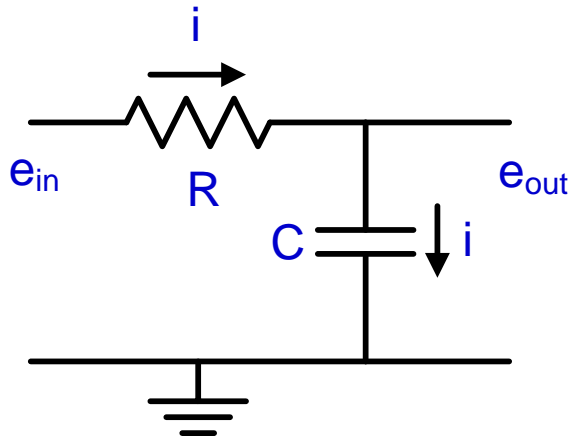
Electrical – Mechanical Analogies

$$\text{Resistor } e = Ri \quad \Leftrightarrow \quad \text{Damper } f = Bv$$

$$\text{Inductor } e = L \frac{di}{dt} \quad \Leftrightarrow \quad \text{Mass } f = M \frac{dv}{dt}$$

$$\text{Capacitor } e = \frac{1}{C} \int i \, dt \quad \Leftrightarrow \quad \text{Spring } f = K \int v \, dt$$

RC Electrical System



$$e_{in} - e_R - e_C = 0$$

$$e_{in} - iR - e_{out} = 0$$

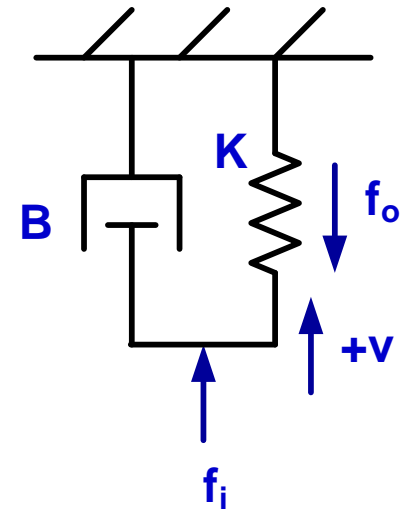
$$e_{in} - \left(C \frac{de_{out}}{dt} \right) R - e_{out} = 0$$

$$RC \frac{de_{out}}{dt} + e_{out} = e_{in}$$

$$\frac{e_{out}}{e_{in}} = \frac{1}{RCD + 1} \quad \tau = RC$$

Analogies

Spring-Damper Mechanical System



$$f_i - f_B - f_K = 0$$

$$f_i - Bv - KX = 0$$

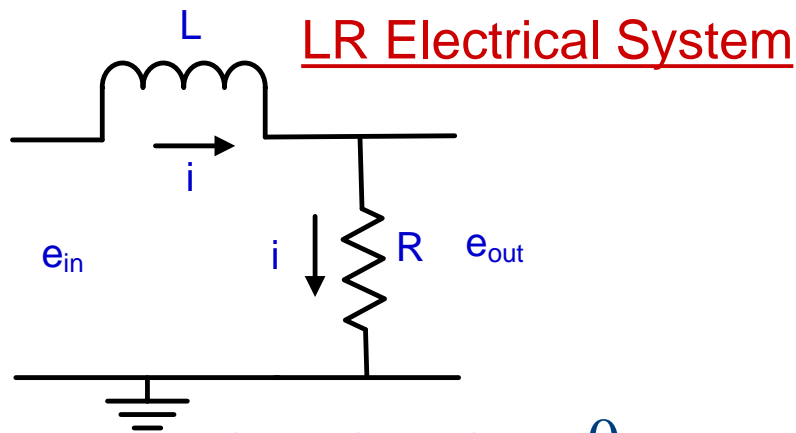
$$f_i - Bv - f_o = 0$$

$$f_i - B \left(\frac{\dot{f}_o}{K} \right) - f_o = 0$$

$$\frac{B}{K} \dot{f}_o + f_o = f_i$$

$$\frac{f_o}{f_i} = \frac{1}{\frac{B}{K} D + 1}$$

$$\tau = \frac{B}{K}$$



$$e_{in} - e_L - e_R = 0$$

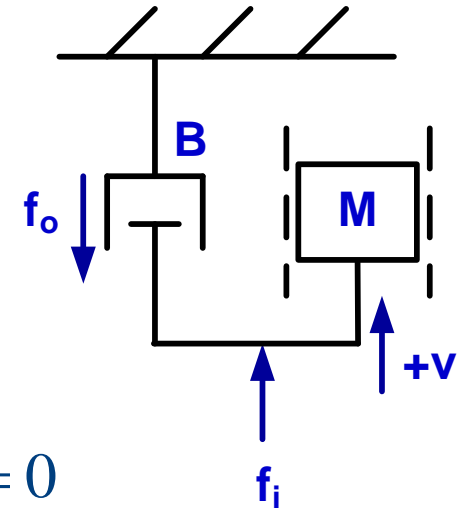
$$e_{in} - L \frac{di}{dt} - e_{out} = 0$$

$$e_{in} - L \frac{d}{dt} \left(\frac{e_{out}}{R} \right) - e_{out} = 0$$

$$\frac{L}{R} \frac{de_{out}}{dt} + e_{out} = e_{in}$$

$$\frac{e_{out}}{e_{in}} = \frac{1}{\frac{L}{R} D + 1} \quad \tau = \frac{L}{R}$$

Mass-Damper Mechanical System



$$f_i - f_B - f_M = 0$$

$$f_i - Bv - M \dot{v} = 0$$

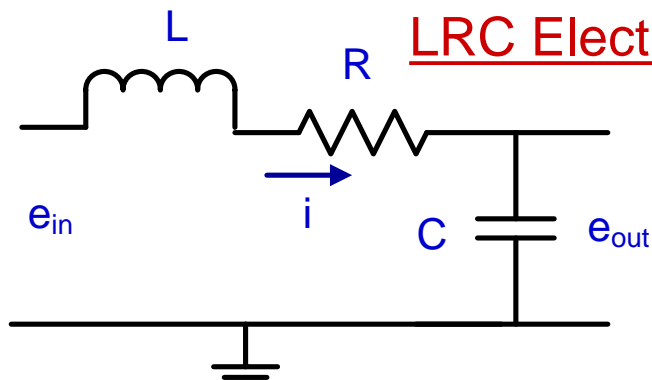
$$f_i - f_o - M \left(\frac{\dot{f}_o}{B} \right) = 0$$

$$\frac{M}{B} \dot{f}_o + f_o = f_i$$

$$\frac{f_o}{f_i} = \frac{1}{\frac{M}{B} D + 1}$$

$$\tau = \frac{M}{B}$$

LRC Electrical System



$$e_{in} - e_L - e_R - e_C = 0$$

$$e_{in} - L \frac{di}{dt} - Ri - e_{out} = 0$$

$$e_{in} - L \frac{d}{dt} \left(C \frac{de_{out}}{dt} \right) - R \left(C \frac{de_{out}}{dt} \right) - e_{out} = 0$$

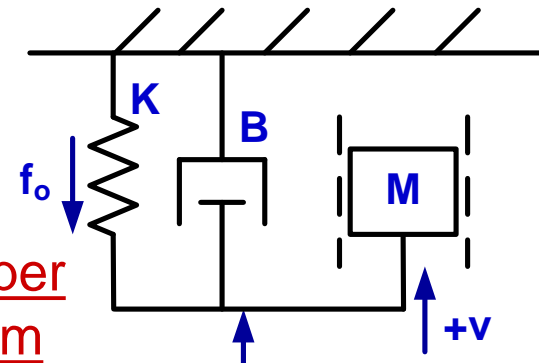
$$LC \frac{d^2 e_{out}}{dt^2} + RC \frac{de_{out}}{dt} + e_{out} = e_{in}$$

$$\frac{e_{out}}{e_{in}} = \frac{1}{LCD^2 + RCD + 1} = \frac{K_S}{\frac{1}{\omega_n^2} D^2 + \frac{2\zeta}{\omega_n} D + 1}$$

$$\omega_n = \sqrt{\frac{1}{LC}} \quad \zeta = \frac{R}{2} \sqrt{\frac{C}{L}} \quad K_S = 1$$

Analogies

Mass-Spring-Damper Mechanical System



$$f_i - f_K - f_B - f_M = 0$$

$$f_i - Kx - Bv - M\dot{v} = 0$$

$$f_i - f_o - B \left(\frac{\dot{f}_o}{K} \right) - M \left(\frac{\ddot{f}_o}{K} \right) = 0$$

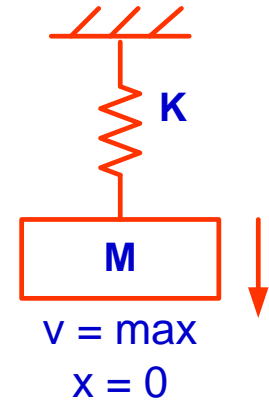
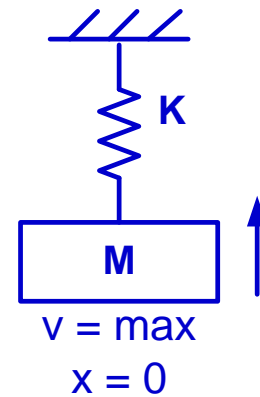
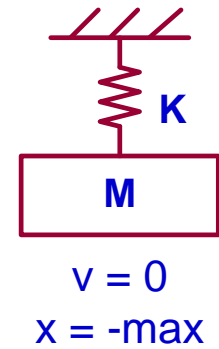
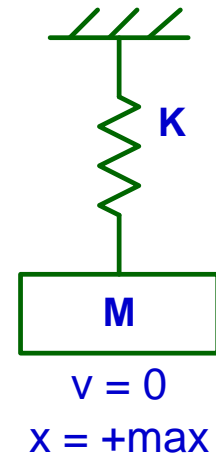
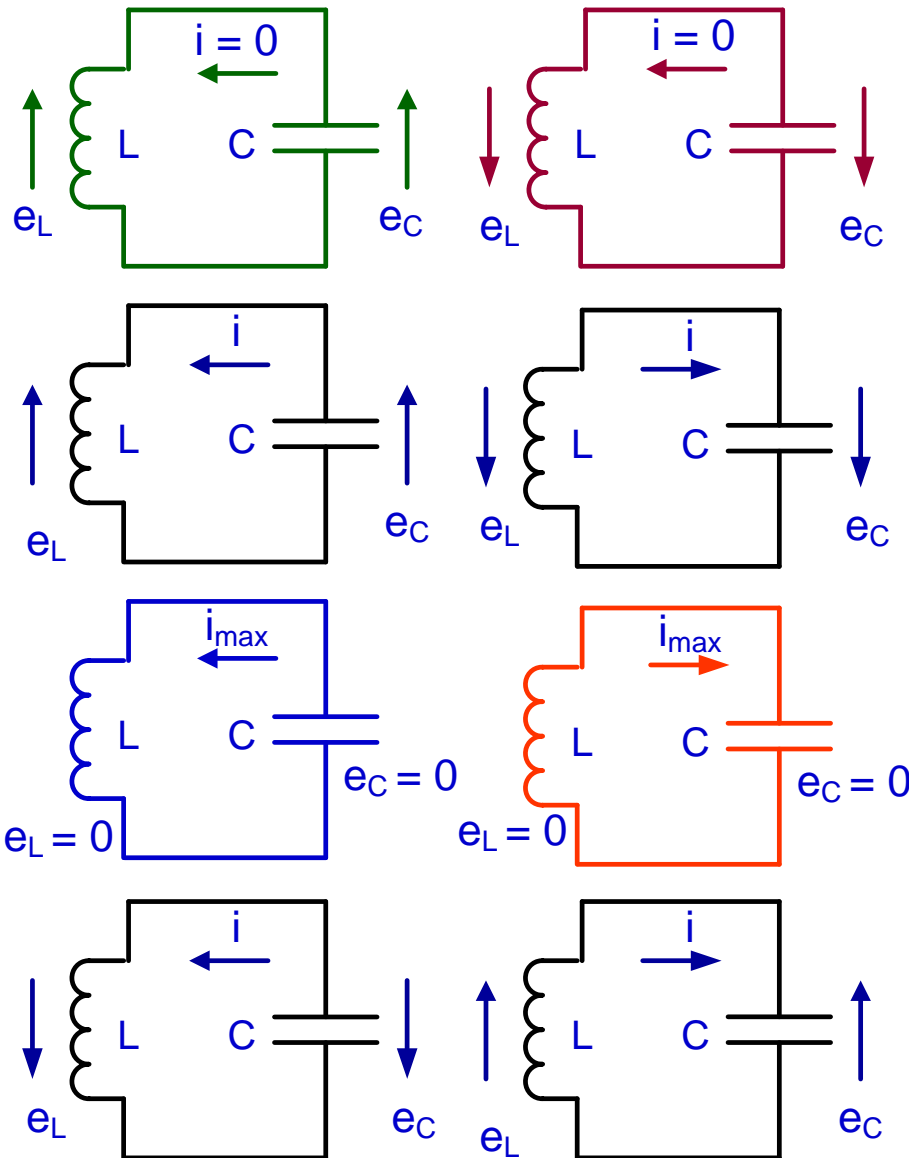
$$\frac{M}{K} \ddot{f}_o + \frac{B}{K} \dot{f}_o + f_o = f_i$$

$$\frac{f_o}{f_i} = \frac{1}{\frac{M}{K} D^2 + \frac{B}{K} D + 1} = \frac{K_S}{\frac{1}{\omega_n^2} D^2 + \frac{2\zeta}{\omega_n} D + 1}$$

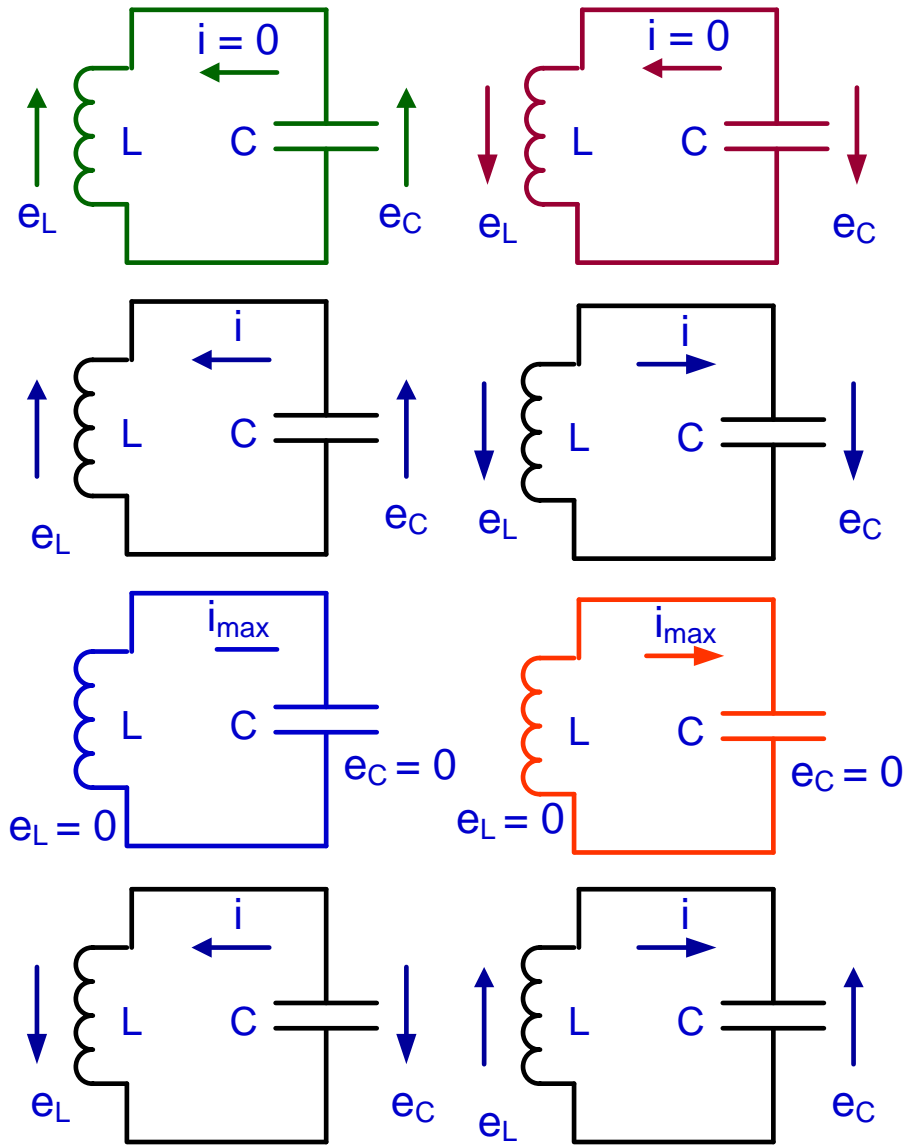
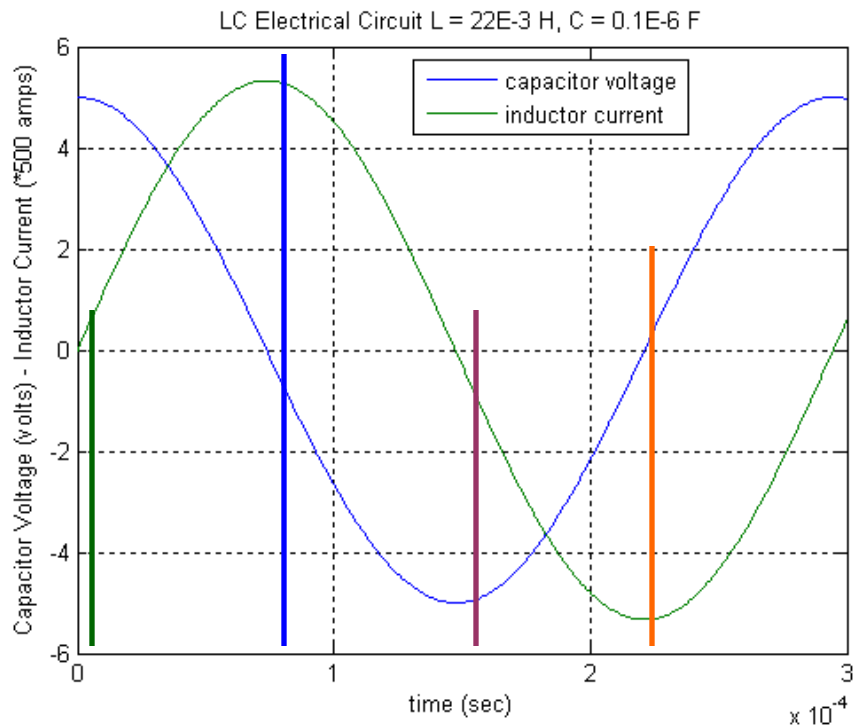
$$\omega_n = \sqrt{\frac{K}{M}} \quad \zeta = \frac{B}{2} \sqrt{\frac{1}{KM}} \quad K_S = 1$$

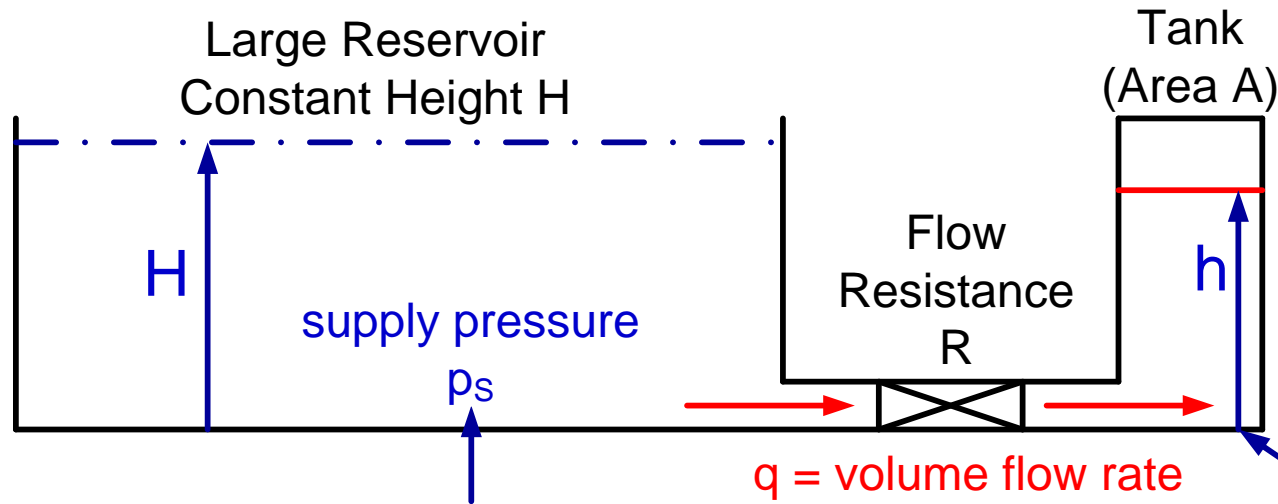
- We can use this analogy to explain the flow of current and the changes in voltages in a LC (inductor-capacitor) electrical circuit – difficult to envision for most mechanical engineers and even for some electrical engineers – by comparing it to a spring-mass mechanical system.
 - The diagrams on the next two slides are color-coded: green, blue, purple, and orange diagrams for each system correspond to each other, as do the vertical lines on the graph indicating capacitor voltage and inductor current at the four specific instances. By comparing the motion of the mass – its changing potential energy corresponding to energy stored in the electric field of the capacitor and its changing kinetic energy corresponding to energy stored in the magnetic field of the inductor – one can better understand how electrical capacitors and inductors function.
- For enhanced multidisciplinary engineering system design and better communication and insight among the design team members, the use of analogies is a powerful addition to an engineer's toolbox.

Inductor-Capacitor (LC) ↔ Mass-Spring (MK) Oscillations



Analogies





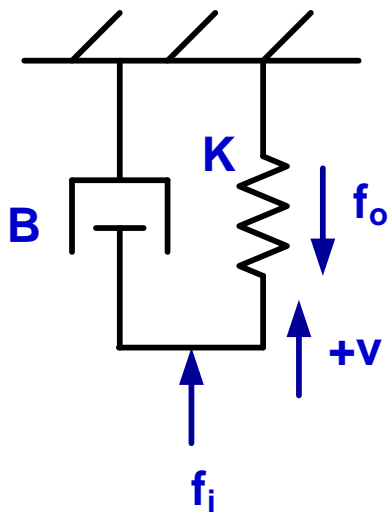
Analogies

$$p_s = \text{constant} = \rho g H \quad (\text{bottom of reservoir})$$

$$\rho = \text{fluid density}$$

$$g = \text{acceleration due to gravity}$$

$$C = \frac{A}{\rho g}$$



$$\frac{B}{K} \frac{df_o}{dt} + f_o = f_i$$

$$RC \frac{dh}{dt} + h = H$$

$$\tau \frac{de_{\text{out}}}{dt} + e_{\text{out}} = K e_{\text{in}}$$