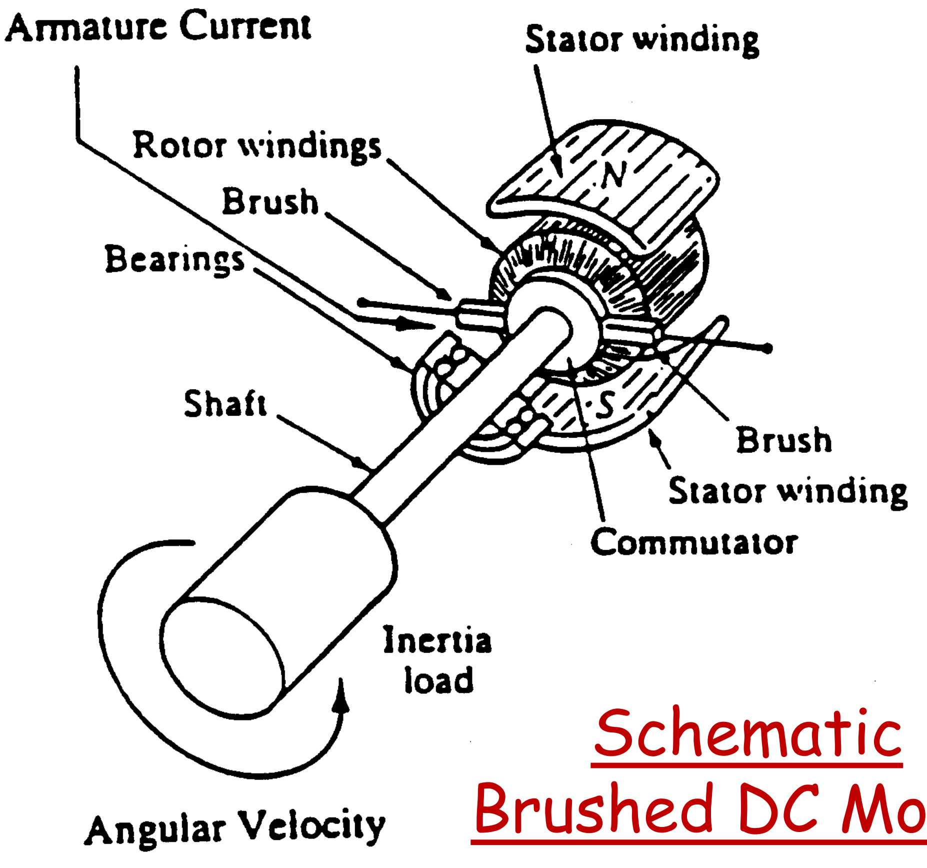
Brushed DC Motor System



Schematic
Brushed DC Motor

Pittman DC
Servo Motor

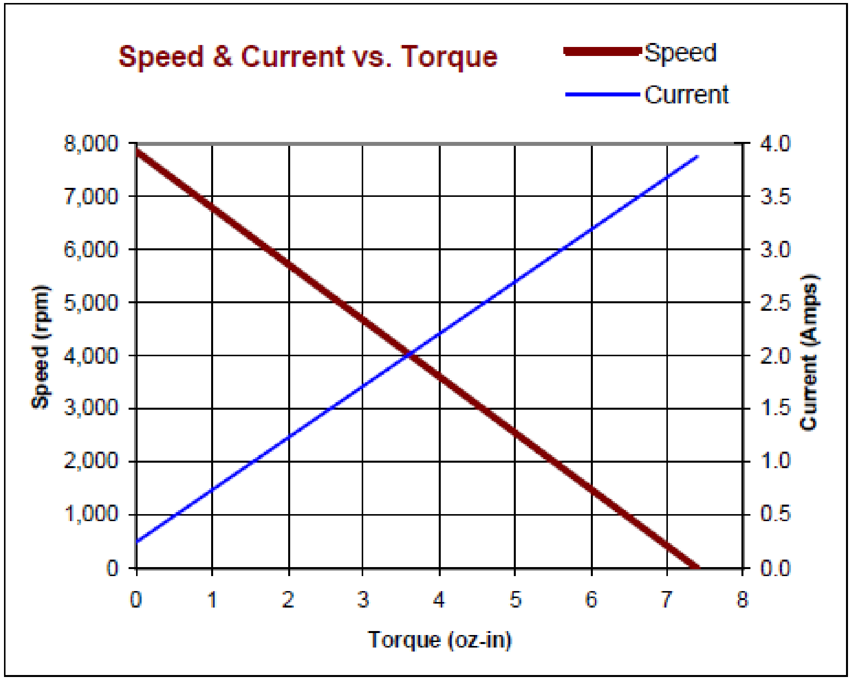
Topics

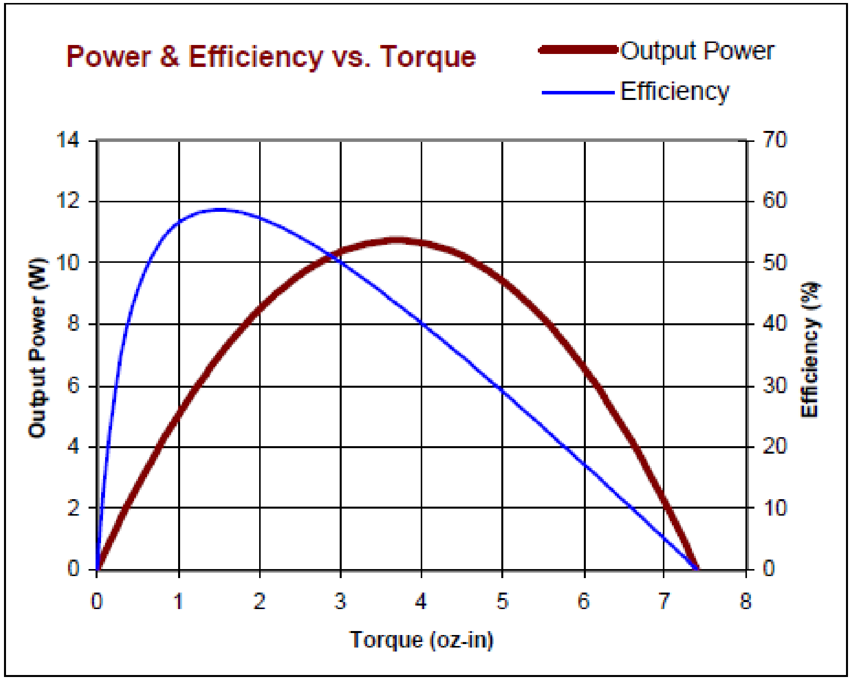
- Brushed DC Motor
  - Physical & Mathematical Modeling
  - Hardware Parameters
  - Model – Hardware Correlation
- H-Bridge Operation

Pittman DC Servo Motor 8322S001

Assembly Data	Symbol	Units	Value	
Reference Voltage	E	V	12	
No-Load Speed	S_{NL}	rpm (rad/s)	7,847	(822)
Continuous Torque (Max.)[1]	T_C	oz-in (N-m)	1.6	(1.1E-02)
Peak Torque (Stall)[2]	T_{PK}	oz-in (N-m)	7.4	(5.2E-02)
Weight	W_M	oz (g)	7.7	(218)
Motor Data				
Torque Constant	K_T	oz-in/A (N-m/A)	1.94	(1.37E-02)
Back-EMF Constant	K_E	V/krpm (V/rad/s)	1.43	(1.37E-02)
Resistance	R_T	Ω	3.10	
Inductance	L	mH	1.57	
No-Load Current	I_{NL}	A	0.25	
Peak Current (Stall)[2]	I_P	A	3.88	
Motor Constant	K_M	oz-in/√W (N-m/√W)	1.12	(7.91E-03)
Friction Torque	T_F	oz-in (N-m)	0.35	(2.5E-03)
Rotor Inertia	J_M	oz-in-s^2 (kg-m^2)	1.4E-04	(9.9E-07)
Electrical Time Constant	τ_E	ms	0.52	
Mechanical Time Constant	τ_M	ms	15.6	
Viscous Damping	D	oz-in/krpm (N-m-s)	0.015	(1.0E-06)
Damping Constant	K_D	oz-in/krpm (N-m-s)	0.92	(6.2E-05)
Maximum Winding Temperature	θ_{MAX}	ºF (ºC)	311	(155)
Thermal Impedance	R_{TH}	ºF/watt (ºC/watt)	75.9	(24.4)
Thermal Time Constant	τ_{TH}	min	7.8	
Encoder Data				
Channels			3	
Resolution		CPR	500	

Pittman DC Servo Motor 8322S001

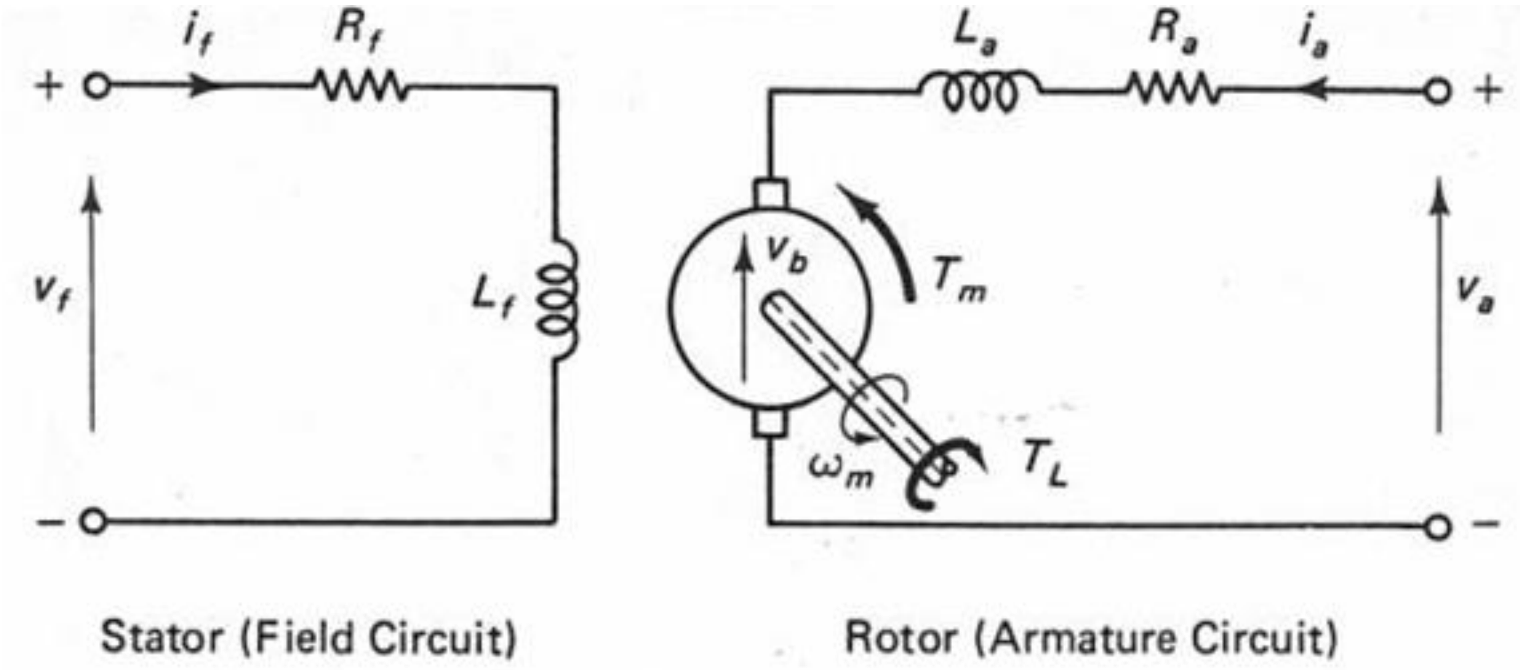




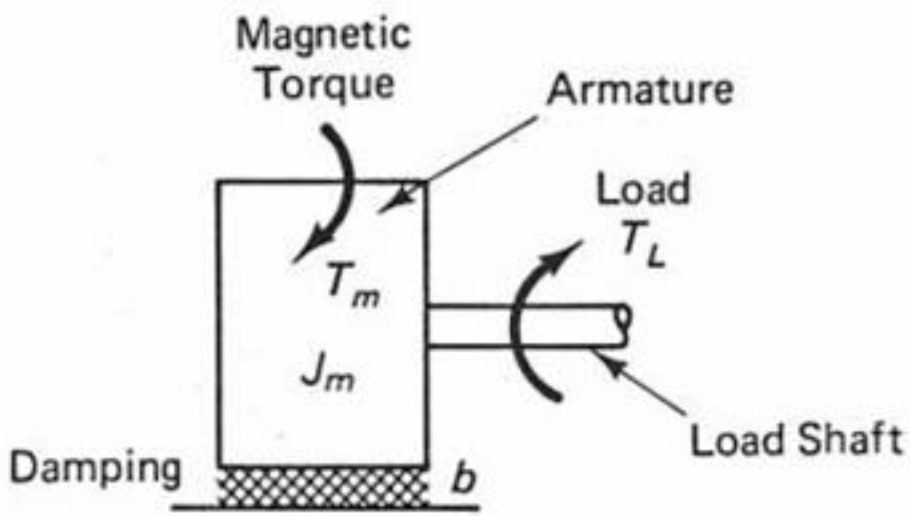
Encoder

500 counts/rev

Wire	Function	Color	Pins
1	GND	Black	GND
2	Index	Green	-
3	CH A	Yellow	
4	Vcc	Red	5V
5	CH B	Blue	



For a permanent-magnet DC motor i_f = constant.



Physical Modeling

- Physical Modeling Assumptions
  - The copper armature windings in the motor are treated as a resistance and inductance in series. The distributed inductance and resistance is lumped into two characteristic quantities, L and R.
  - The commutation of the motor is neglected. The system is treated as a single electrical network which is continuously energized.
  - The compliance of the shaft connecting the load to the motor is negligible. The shaft is treated as a rigid member.
  - The total inertia J is a single lumped inertia, equal to the sum of the inertias of the rotor and the driven load.

- There exists motion only about the axis of rotation of the motor, i.e., a one-degree-of-freedom system.
- The parameters of the system are constant, i.e., they do not change over time.
- The damping in the mechanical system is modeled as viscous damping B, i.e., all stiction and dry friction are initially neglected.
- The optical encoder output is decoded in software. Position and velocity are calculated and made available as analog signals for control calculations. The motor is driven with a PWM control signal to a H-Bridge. The time delay associated with this, as well as computation for control, is lumped into a single system time delay.

- <u>Mathematical Modeling Steps</u>
  - Define System, System Boundary, System Inputs and Outputs
  - Define Through and Across Variables
  - Write Physical Relations for Each Element
  - Write System Relations of Equilibrium and/or Compatibility
  - Combine System Relations and Physical Relations to Generate the Mathematical Model for the System

Physical Relations

$$V_L = L\frac{di_L}{dt} \qquad V_R = Ri_R \qquad T_B = B\omega$$

$$T_J = J\alpha = J\dot{\omega} \qquad J \equiv J_{motor} + J_{load}$$

$$T_m = K_t i_m \qquad V_b = K_b \omega$$

$$P_{out} = T_m \omega = K_t i_m \omega \qquad P_{in} = V_b i_m = K_b \omega i_m$$

$$\frac{P_{out}}{P_{in}} = \frac{K_t}{K_b}$$

$$P_{out} = P_{in}$$

$$K_t = K_b \equiv K_m$$

$$\begin{cases} K_t(oz-in/A) = 1.3524 K_b(V/krpm) \\ K_t(Nm/A) = 9.5493 \times 10^{-3} K_b(V/krpm) \\ K_t(Nm/A) = K_b(V-s/rad) \end{cases}$$

System Relations + Equations of Motion

$$V_{in} - V_R - V_L - V_b = 0$$

$$T_m - T_B - T_J = 0$$

$$i_R = i_L = i_m \equiv i$$

KVL

$$V_{in} - Ri - L\frac{di}{dt} - K_b\omega = 0$$

Newton's Law

$$J\frac{d\omega}{dt} + B\omega - K_t i = 0$$

$$\begin{bmatrix} \dfrac{d\omega}{dt} \\ \dfrac{di}{dt} \end{bmatrix} = \begin{bmatrix} -B/J & K_t/J \\ -K_b/L & -R/L \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_{in}$$

Steady-State Conditions

$$V_{in} - Ri - L\frac{di}{dt} - K_b\omega = 0$$

$$V_{in} - R\left(\frac{T}{K_t}\right) - K_b\omega = 0$$

$$T = \frac{K_t}{R}V_{in} - \frac{K_t K_b}{R}\omega$$

$$T_s = \frac{K_t}{R}V_{in}$$ Stall Torque

$$\omega_0 = \frac{V_{in}}{K_b}$$ No-Load Speed

Transfer Functions

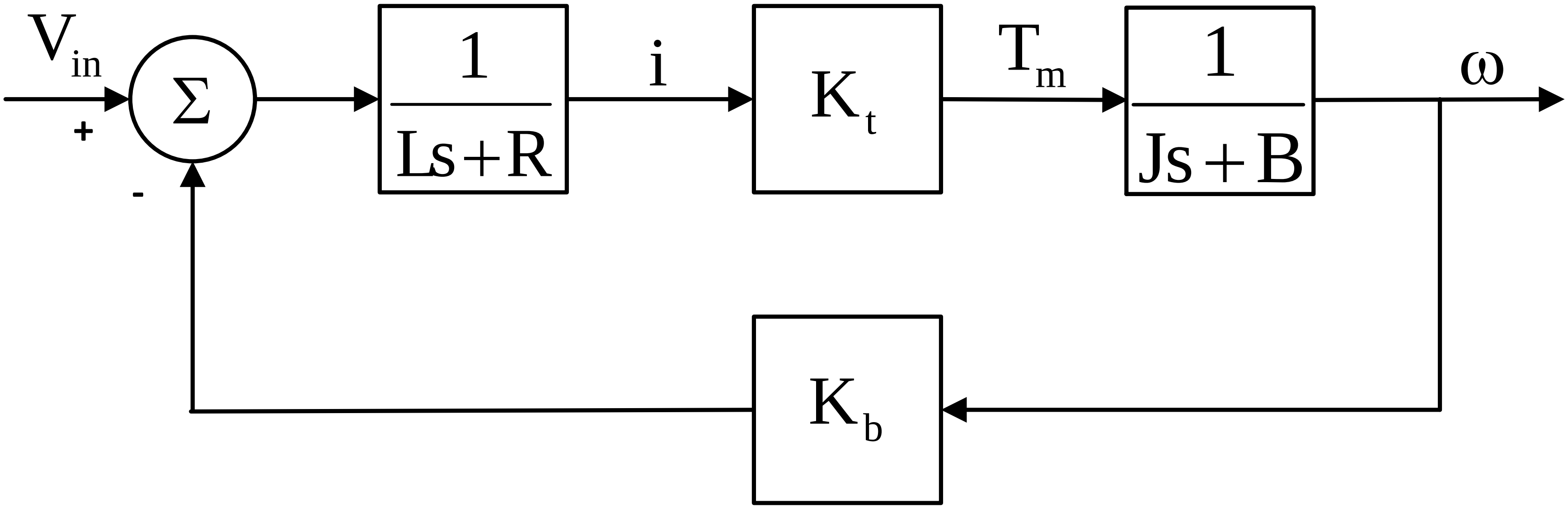
$$V_{in} - Ri - L\frac{di}{dt} - K_b\omega = 0 \qquad J\frac{d\omega}{dt} + B\omega - K_t i = 0$$

$$V_{in}(s) - (Ls + R)I(s) - K_b\Omega(s) = 0 \qquad (Js + B)\Omega(s) - K_t I(s) = 0$$

$$\frac{\Omega(s)}{V_{in}(s)} = \frac{K_t}{(Js + B)(Ls + R) + K_t K_b} = \frac{K_t}{JLs^2 + (BL + JR)s + (BR + K_t K_b)}$$

$$= \frac{\frac{K_t}{JL}}{s^2 + \left(\frac{B}{J} + \frac{R}{L}\right)s + \left(\frac{BR}{JL} + \frac{K_t K_b}{JL}\right)}$$

Block Diagram



Simplification

$$\tau_m = \frac{J}{B} \gg \tau_e = \frac{L}{R}$$

$$V_{in} - Ri - K_b\omega = 0 \qquad J\frac{d\omega}{dt} + B\omega - K_t i = 0$$

$$J\frac{d\omega}{dt} + B\omega = K_t i = K_t\left(\frac{1}{R}\left(V_{in} - K_b\omega\right)\right) = \frac{K_t}{R}\left(V_{in} - K_b\omega\right)$$

$$\frac{d\omega}{dt} + \left(\frac{K_t K_b}{RJ} + \frac{B}{J}\right)\omega = \frac{K_t}{RJ}V_{in}$$

$$\frac{d\omega}{dt} + \left(\frac{1}{\tau_{motor}} + \frac{1}{\tau_m}\right)\omega = \frac{K_t}{RJ}V_{in}$$

$$\frac{d\omega}{dt} + \left(\frac{1}{\tau_{motor}}\right)\omega = \frac{K_t}{RJ}V_{in} \qquad \text{since } \tau_m \gg \tau_{motor}$$

Pitmann Brushed DC Motor Modeling:

Equations of Motion

$$e_{in} = L\frac{di}{dt} + Ri + K_b\omega$$

$$J\dot{\omega} + B\omega = K_t i$$

Coulomb friction neglected

Open-Loop Transfer Function $\omega / e_{in}(D)$:

$$\frac{\omega}{e_{in}} = \frac{K_t / JL}{D^2 + \left(\frac{B}{J} + \frac{R}{L}\right)D + \left(\frac{BR}{JL} + \frac{K_t K_b}{JL}\right)}$$

$$\left.\begin{array}{l}\tau_m = J/B \\ \tau_e = L/R\end{array}\right\} \quad \tau_m \gg \tau_e$$

$$J\dot{\omega} + B\omega = K_t i = K_t\left[\frac{1}{R}(e_{in} - K_b\omega)\right] = \frac{K_t}{R}(e_{in} - K_b\omega)$$

$\tau_{motor} = \frac{RJ}{K_t K_b}$

$$\dot{\omega} + \left(\frac{K_t K_b}{RJ} + \frac{B}{J}\right)\omega = \frac{K_t}{RJ}e_{in}$$

$\tau_m \gg \tau_{motor}$

$$\dot{\omega} + \left(\frac{1}{\tau_{motor}} + \frac{1}{\tau_m}\right)\omega = \frac{K_t}{RJ}e_{in} \Rightarrow \dot{\omega} + \left(\frac{1}{\tau_{motor}}\right)\omega = \frac{K_t}{RJ}e_{in}$$

Analysis: Pittman 83225001 Brushed DC Servo Motor

$J_{motor} = 9.9 \times 10^{-7}$ kg-m^2

$J_{load} = \frac{1}{2} m R^2 \qquad m = (\pi R^2)(t)(\rho_{brass})$

$= 9.93 \times 10^{-6}$ kg-m^2

$\rho_{brass} = 8500 \ \frac{kg}{m^3}$

$t = .25'' = .00635 \ m$

$R = .0185 \ m$

$\frac{J_L}{J_m} = 10.03 \qquad J_{total} = 10.92 \times 10^{-6}$ kg-m^2

$B = 1.0 \times 10^{-6}$ N-m-s

$K_t = 1.37 \times 10^{-2}$ (N-m)/A

$K_b = 1.37 \times 10^{-2}$ V/rad/s

$R = 3.10 \ \Omega$

$L = 1.57 \times 10^{-3}$ H

$\tau_m = J/B = 10.92$

$\tau_e = L/R = 5.06 \times 10^{-4}$

$\tau_m \gg \tau_e$

$\frac{\tau_m}{\tau_e} = 2.16 \times 10^4$

$\tau_{motor} = \frac{RJ}{K_t K_b} = 0.180$

$\tau_m / \tau_{motor} = 60.5$

$$\tau_{motor} \dot{\omega} + \omega = \frac{1}{K_b} e_{in}$$

$$\tau_{motor}\dot{\omega} + \omega = \frac{1}{K_b} e_{in}$$

$$(0.180)\dot{\omega} + \omega = (73.0) e_{in}$$

$$\tau\dot{\omega} + \omega = K e_{in}$$ 1^{ST}-Order ODE

$$\frac{\omega}{e_{in}} = \frac{K}{\tau D + 1}$$

$\left\{ \begin{array}{l} \tau = 0.180 \text{ sec} \\ K = 73.0 \end{array} \right.$ verify by experiment.

From Pittman Data Sheet

Motor Constant $K_M = 7.91 \times 10^{-3} = \frac{K_t}{\sqrt{R}}$ $\frac{N\text{-}m}{\sqrt{W}}$ W = watts $(i^2 R)$

$T_f = 2.5 \times 10^{-3}$ N-m friction Torque (Coulomb friction dynamic)

$\tau_E = 0.52 \times 10^{-3}$ sec $= L/R$ same as τ_e

$\tau_M = 15.6 \times 10^{-3}$ sec $= \frac{RJ}{K_t K_b}$ same as τ_{motor} (note: $J = J_m$ on data sheet)

K_D = damping constant $= \frac{K_t K_b}{R} = 6.2 \times 10^{-5}$ N-m-s (same units as B)

Steady-State Analysis

$$e_{in} = L\frac{di}{dt} + Ri + K_b\omega \implies e_{in} = Ri + K_b\omega$$

$$J\dot{\omega} + B\omega = K_t i \implies B\omega = K_t i$$

Define $T = B\omega + T_f + T_L$ (includes all load torque)

$$e_{in} - Ri - K_b\omega = 0$$

$$e_{in} - R\left(T/K_t\right) - K_b\omega = 0 \implies T = \frac{K_t}{R}e_{in} - \frac{K_t K_b}{R}\omega$$

$$= K_M\sqrt{R}\,e_{in} - K_D\,\omega$$

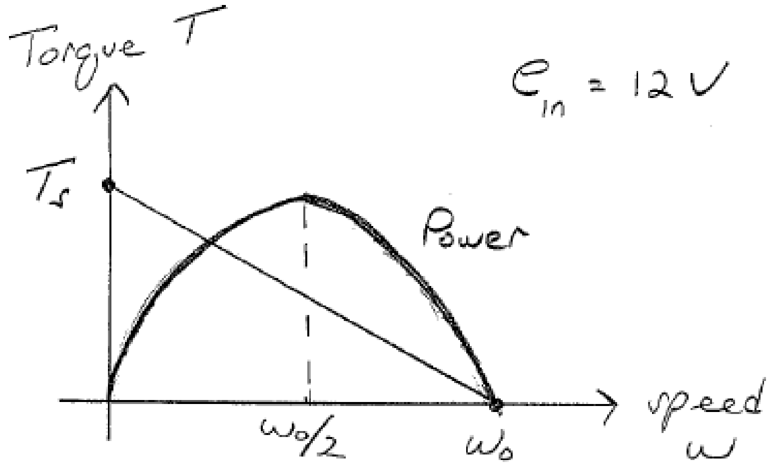
(Pittman parameters)

$$T = \frac{K_t}{R}e_{in} - \frac{K_t K_b}{R}\omega$$

Linear Torque-Speed Relation

$\omega = 0 \implies T_s = \frac{K_t}{R}e_{in}$ stall torque

$T = 0 \implies \omega_0 = e_{in}/K_b$ no-load speed



$e_{in} = 12\ V$

$$\begin{cases} T_s = \dfrac{K_t}{R} e_{in} = 0.053\ \text{N-m} \\ \omega_0 = e_{in}/K_b = 876\ \text{rad/s} \end{cases}$$

(822 in data sheet)

$$\text{Power } P = T\omega = \omega\left[\frac{K_t}{R} e_{in} - \frac{K_t K_b}{R}\omega\right]$$

$$= \omega\left[T_s - \frac{T_s}{\omega_0}\omega\right] = \omega T_s\left[1 - \frac{\omega}{\omega_0}\right]$$

$$P = T_s\left[\omega - \frac{\omega^2}{\omega_0}\right]$$

$$\frac{dP}{d\omega} = T_s - \frac{2T_s}{\omega_0}\omega = 0 \implies \omega_{max} = \frac{\omega_0}{2}$$

Peak Current (stall) $= e_{in}/R = 3.87\ A$

No-Load Current $= 0.25\ A$

Pittman Data Sheet:

no-load speed = 822 rad/s
no-load current = 0.25 A

Interpret this information

no-load current is a measure of friction losses.

no-load current = 0.25 A

$$e_{in} = L\frac{di}{dt} + Ri + K_b\omega \qquad \frac{di}{dt} = 0$$

no-load speed

$$\omega = \frac{e_{in} - Ri}{K_b} = \frac{12 - (3.10)(0.25)}{(1.37\times10^{-2})} = 819.3 \text{ rad/s}$$

(close to 822 given)

at this no-load speed

$$T_m = K_t i = (1.37\times10^{-2})(.25) = 3.43\times10^{-3} \text{ N-m}$$

at steady-state

$$T_m = T_{viscous} + T_{coulomb}$$

$$= B\omega + T_f$$

$$= (1.0\times10^{-6})(819.3) + (2.5\times10^{-3}) = 3.32\times10^{-3} \text{ N-m}$$

Load Torque vs. Efficiency

Rated Armature Voltage = 12 V

Load Torque T_L = 1.5 oz-in = $(1.5\text{ oz-in})\left(\frac{1\text{N}}{3.6\text{ oz}}\right)\left(\frac{1\text{m}}{39.37''}\right) =$ 1.06×10^{-2} N-m

$B = 1.0\times10^{-6}$ N-m-s

$K_t = 1.37\times10^{-2}\ \frac{\text{N-m}}{\text{A}}$

$T_f = 2.5\times10^{-3}$ N-m

During steady state

$$K_t i = B\omega + T_f + T_L$$

$$\omega = \frac{1}{B}\left(K_t i - T_f - T_L\right)$$

substitution

Also during steady state:

$$e = Ri + K_b\omega$$

$$i = \frac{e}{R} - \frac{K_b\omega}{R} = \frac{e}{R} - \frac{K_b}{RB}\left(K_t i - T_f - T_L\right)$$

Solve for i: $$i = \frac{e + (K_b/B)(T_f + T_L)}{R + \frac{K_b K_t}{B}}$$

Substitute numbers:

$$i = \frac{1}{190.8}\left[12 + (1.37\times10^{4})(2.5\times10^{-3} + T_L)\right] = 1.004\text{ A}$$

$$\omega = \frac{1}{1.0\times10^{-6}}\left[(1.37\times10^{-2})(i) - (2.5\times10^{-3}) - T_L\right] = 654.8\text{ rad/s}$$

Power Input : $P_{in} = e i = (12)(1.004) = 12.05$ W

Power Output : $P_{out} = T_L \omega = (1.06 \times 10^{-2})(654.8) = 6.94$ W

$$\text{Efficiency } \% = \frac{P_{out}}{P_{in}}(100) = \frac{6.94}{12.05}(100) = 57.6\ \%$$

Matches Pittman Graph ✓

Losses : $P_{i^2R} = R i^2 = (3.10)(1.004)^2$

$= 3.125$ W

$P_{friction} = (B\omega + T_F)\omega = \left[(1.0 \times 10^{-6})(654.8) + (2.5 \times 10^{-3})\right] \times (654.8)$

$= 2.066$ W

Total Losses : $2.066 + 3.125 = 5.191$ W

Power Input = Power Output + P_{Losses}

$12.05 = 6.94 + 5.19$

$12.05 \approx 12.13$ ✓

% Pittman Motor 8322S001 Load Torque vs. Efficiency Plot

MatLab M-File

```
B = 1.0E-6;
Kt = 1.37E-2;
Tf = 2.5E-3;
Kb = 1.37E-2;
R = 3.10;
TL = 0:.1:7.5;
TL = TL/3.6/39.37;
e = 12;
i = (1/(R + Kb*Kt/B))*(e + (Kb/B)*(Tf + TL));
omega = (1/B)*((Kt*i)- Tf - TL);
Pin = e*i;
Pout = TL.*omega;
E = (Pout./Pin)*100;
plot(TL*3.6*39.37,E)
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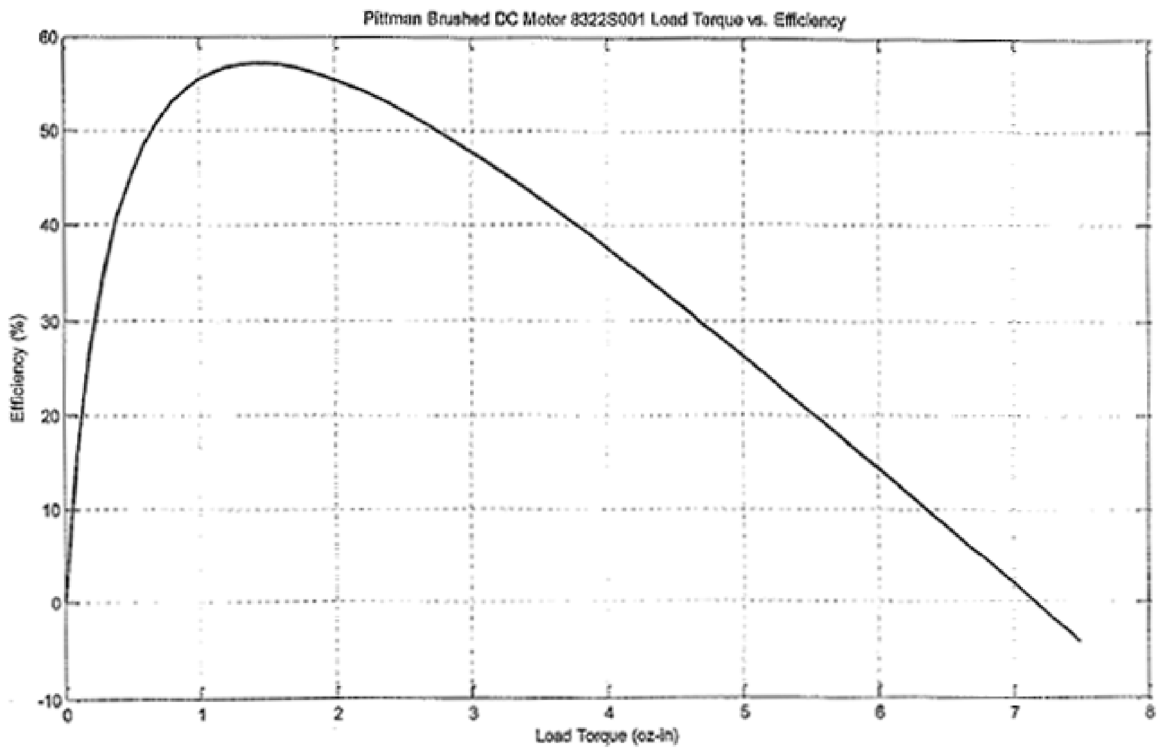
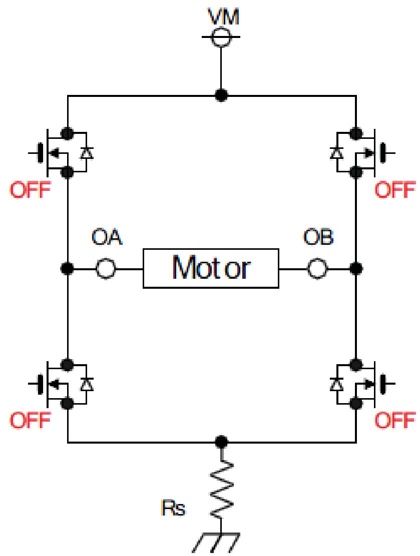


H-Bridge Operation

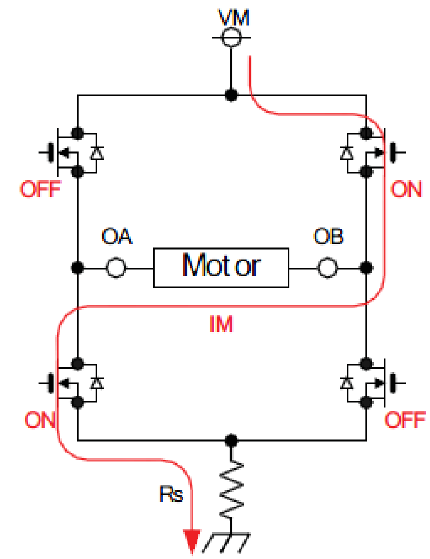
- For DC electric motors, a power device configuration called an H-Bridge is used to control the direction and magnitude of the voltage applied to the load. The H-Bridge consists of four electronic power components arranged in an H-shape in which two or none of the power devices are turned on simultaneously.
- A typical technique to control the power components is via a PWM (Pulse Width Modulation) signal. A PWM signal has a constant frequency called the *carrier frequency*. Although the frequency of a PWM signal is constant, the width of the pulses (the duty cycle) varies to obtain the desired voltage to be applied to the load.

- The H-Bridge can be in one of the four states: coasting, moving forward, moving backward, or braking, as shown on the next slide.
  - In the coasting mode, all four devices are turned off and since no energy is applied to the motor, it will coast.
  - In the forward direction, two power components are turned on, one connected to the power supply and one connected to ground.
  - In reverse direction, only the opposite power components are turned on supplying voltage in the opposite direction and allowing the motor to reverse direction.
  - In braking, only the two devices connected to ground are tuned on. This allows the energy of the motor to quickly dissipate, which will take the motor to a stop.

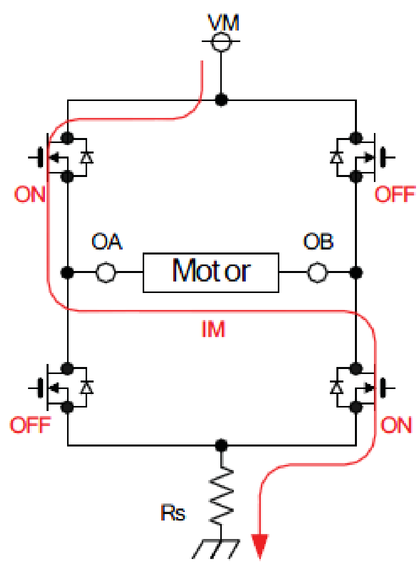
(A) Free (coast)



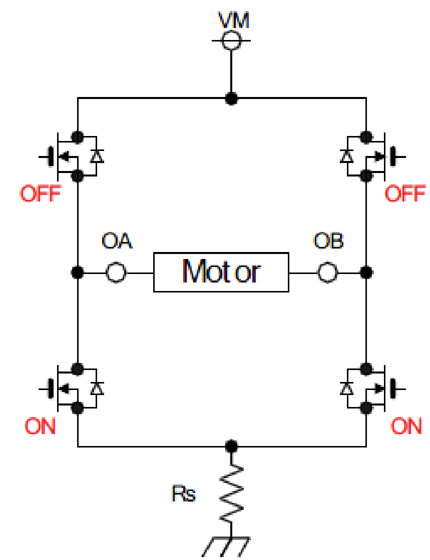
(C) Reverse



(B) Forward



(D) Brake



- The four diodes shown in anti-parallel to the transistors are for back-EMF current decay when all transistors are turned off.
- These diodes protect the transistors from the voltage spike on the motor leads due to the back-EMF when all four transistors are turned off. This could yield excessive voltage on the transistor terminals and potentially damage them.
- They must be sized to a current higher than the motor current and for the lowest forward voltage to reduce junction temperature and the time to dissipate the motor energy.

Diodes for back-EMF protection are shown. The solid line is the current flow when the transistors on the upper left corner and on the lower right corner are turned on. The dashed line shows the motor current when all transistors are turned off.