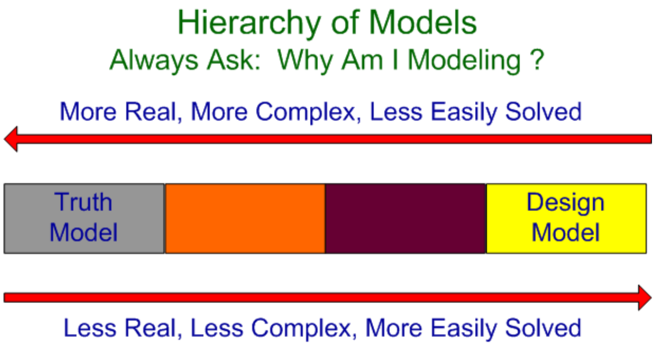
Engineering System Investigation Process: The Cornerstone of Modern Engineering Practice

In design an engineer rarely starts with a blank sheet of paper. Designs are usually the result of the improvement of an existing system, the innovative combination of existing systems, or the application of new technology or new knowledge to an existing system. In all this, understanding what exists is paramount and modeling is essential to that understanding. The purpose of modeling is insight! Also once a concept has been developed in the conceptual phase of design it is evaluated through modeling – not by building and testing – the physical system, sensors, actuators, and controls, all integrated into the design concept. No after-thought add-ons are allowed!

Ask six engineers what a model is and you may get six different answers. The word model has a specific meaning and modeling is the single most important activity in the modern multi-disciplinary engineering system design process. There are actually two distinct models of an actual dynamic physical system: a physical model and a mathematical model, and the distinction between them is most important. In general, a physical model is an imaginary physical system – a slice of reality – and in modeling dynamic physical systems we use engineering judgment and simplifying assumptions to develop a physical model. The challenges to physical modeling are formidable as the dynamic behavior of many physical processes is very complex. There is a hierarchy of physical models of varying complexity possible, from the less-real, less-complex, more-easily-solved design model to the more-real, more-complex, less-easily-solved truth model. The complexity of the physical model depends on the particular need, e.g., system design iteration, control system design, control design verification, physical understanding. Always ask the question: Why am I modeling? An excellent analogy is geographic maps and the varying detail one can display on a map.

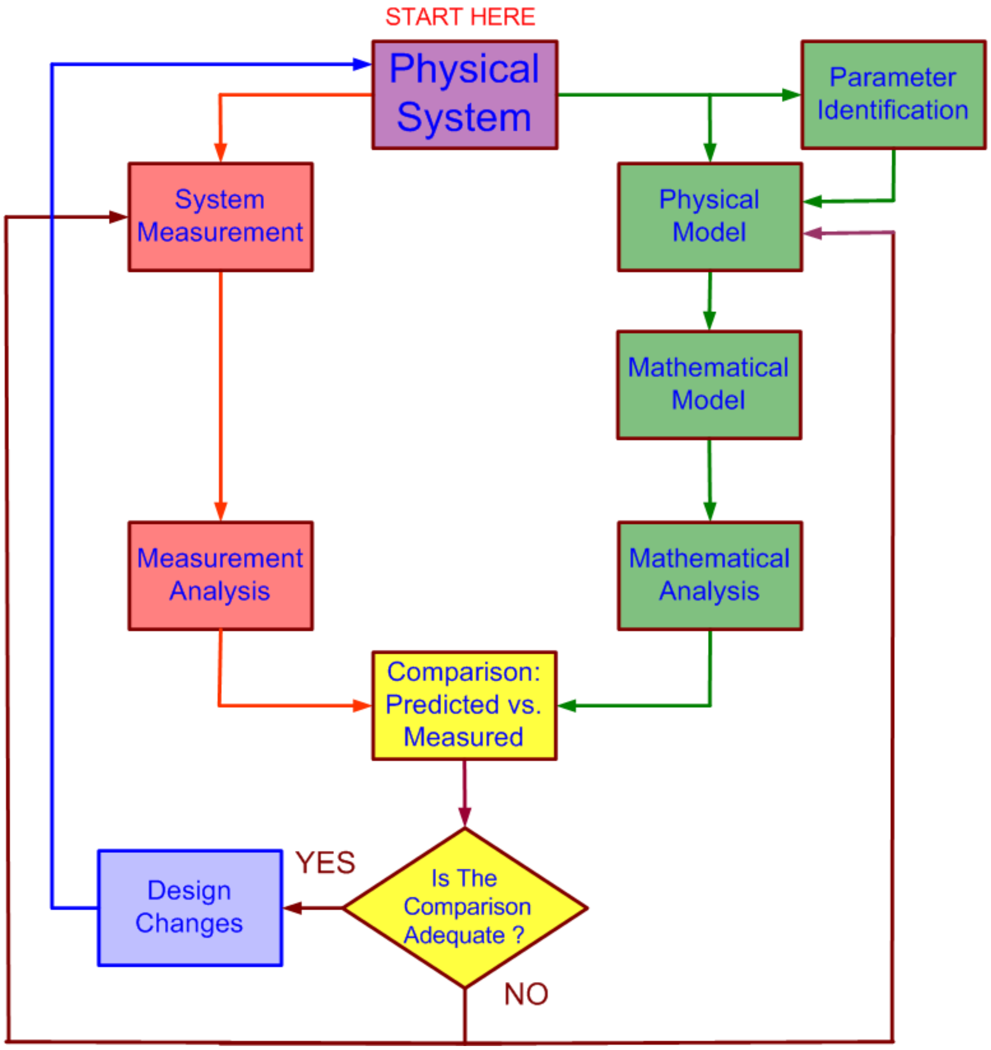


The intelligent use of simple physical models requires that we have some understanding of what we are missing when we choose the simpler model over the more complex model. The astuteness with which simplifying approximations are made at the onset of an investigation is the very crux of engineering analysis. The ability to make shrewd and viable approximations which greatly simplify the system and still lead to a rapid, reasonably accurate prediction of its behavior is the hallmark of every successful engineer. Once a physical model has been developed, the appropriate laws of nature, e.g., Newton's Laws, Maxwell's Equations, Conservation of Mass and Energy, are applied to the physical model to generate the mathematical model, i.e., the differential equations describing the dynamic behavior of the physical model.

The engineering system investigation process is a procedure an engineer follows to thoroughly investigate, i.e., understand, predict, and experimentally verify, how a dynamic engineering system or device performs, no matter how simple or complex the system may be. It is an iterative process, as understanding how the system performs requires simplifying assumptions initially. These initial simplifying assumptions may later be relaxed or changed as understanding develops through comparison of analytical predictions with experimental observations. Comparing the predicted dynamic behavior with the actual measured dynamic behavior is a key step in the investigation process, as prediction without experimental verification is at best, questionable, and at worst, useless. It is important to note that the steps in this process should be applied not only when an actual physical system exists and one desires to understand and predict its behavior, but also when the physical system is a concept in the design process that needs to be analyzed and evaluated. After recognizing a need for a new product or service, one uses past experience (personal and vicarious), awareness of existing hardware, understanding of physical laws, and creativity to generate design concepts. The importance of modeling and analysis in the design process has never been more important. These design concepts can no longer be evaluated by the build-and-test approach because it is too costly and time consuming. Validating the predicted dynamic behavior in this case, when no actual physical system exists, then becomes even more dependent on one's past hardware and experimental experience.

The process is shown in the diagram below. A description of the steps follows.

Engineering System Investigation Process



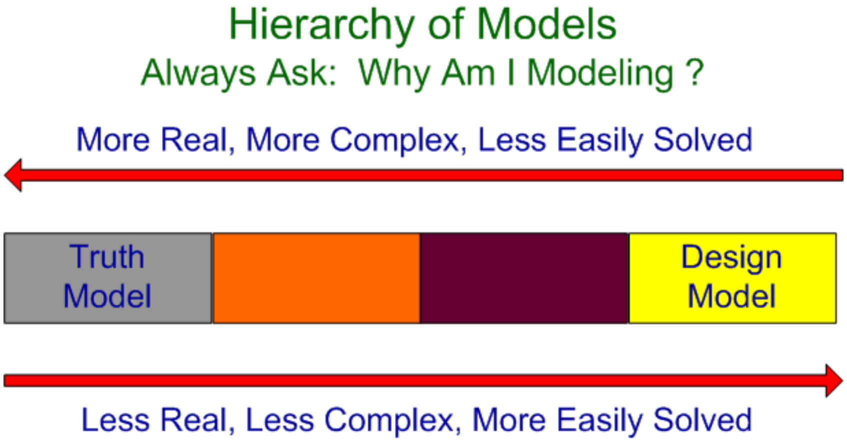
⇒ Physical System

- The process starts with an actual physical system or product. It could be an actual engineered product or device, e.g., a computer hard-disk drive or an artificial organ, or it could be a basic dynamic system used for instructional purposes, e.g., a spring-mass mechanical system or a resistor-capacitor electrical low-pass filter. The physical system must be completely understood. How does it work? What materials does it use? What problem was it designed to solve? What need was it meant to satisfy? Who was the customer? Why was it designed the way it was? Why is it innovative? What alternative designs were considered? Also, when concepts are developed as part of the engineering design process, the physical system could be one of those concepts which needs to be understood and evaluated, not by building and testing it, but through modeling, analysis, and prediction with some experimental verification.

- Let's use as an example of a physical system the spring-mass dynamic system, the simplest mechanical dynamic system one could create. A picture of the system is shown. One finds springs and moving masses in vehicle suspension systems and in automatic machinery of all kinds. But what is most important to understand is that the essential characteristics (springiness, mass, and energy loss) of the spring-mass system are present in almost every mechanical system. We will develop this concept more fully as we proceed.
- The spring-mass system shown consists of a mass hanging at the tip of a tension spring (see picture) that is attached to a stationary support. A tension spring can only be stretched. In its rest state, the spring coils are pulled together against each other so no compression of the spring is possible. The motion of the spring-mass system is constrained by a linear ball-bearing on the side of the support so that the mass oscillates only in one direction, the vertical direction. For measurement purposes, a non-contact optical (infrared) sensor attached to the base is used to measure the position of the mass; the output of the sensor is an electrical voltage proportional to the mass position. The mass is free to oscillate up and down with energy being dissipated mainly by the friction in the bearing and also by the cyclical motion (stretching and relaxing) of the spring. Air resistance has a minimal effect on the motion of the mass. The mass is set in motion by simply displacing it from its static equilibrium position (i.e., its rest position hanging motionless from the spring) and then releasing it.

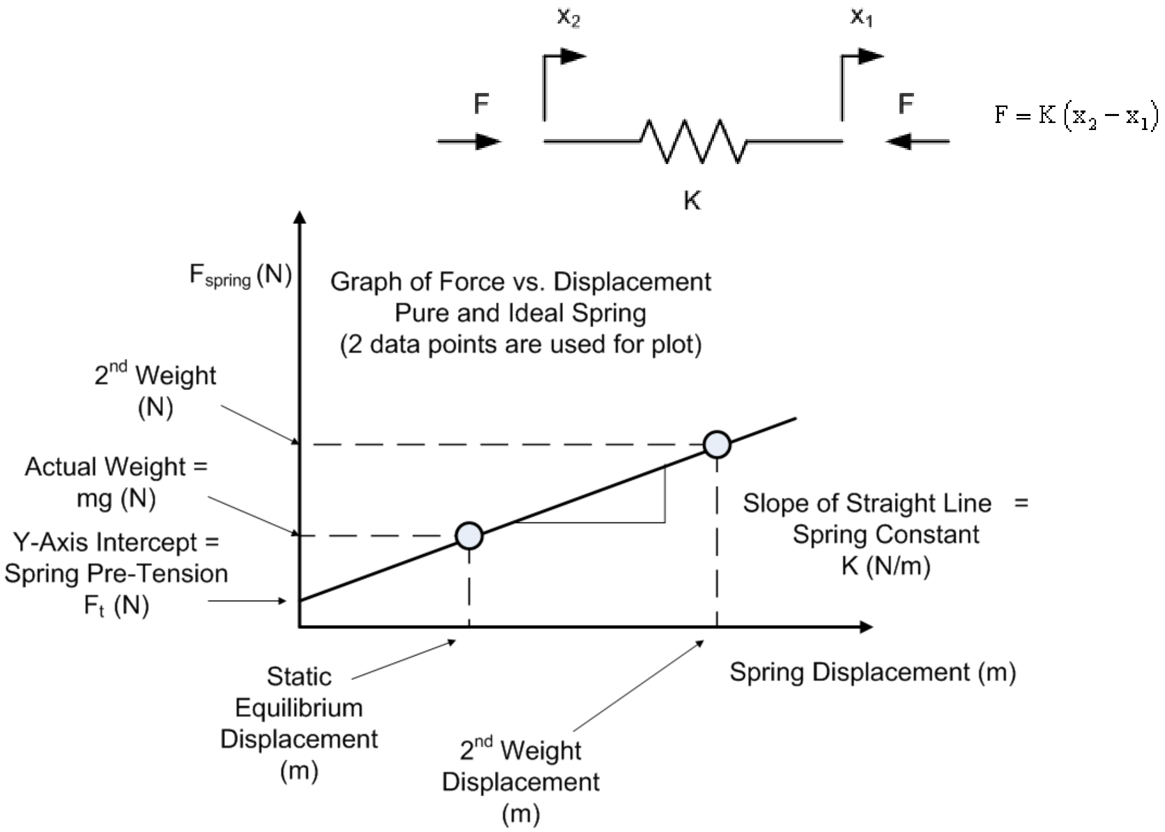
⇒ **Physical Model**

- This step is the key step in the entire process. Unfortunately, it is the least understood. By the use of simplifying assumptions and engineering judgment, learned through much repetition, we develop a physical model, a slice of reality, which is not an actual piece of hardware, but an approximation of the actual system capturing the essential elements of the actual system in as much detail as the need for the model requires. There is a hierarchy of models possible – from the less-complex, less-realistic, more-easily-solved design model to the more-complex, more-realistic, less-easily-solved truth model – depending on the particular need for the model, e.g., design iteration, control system design, final verification before hardware implementation. Always ask the question “Why am I modeling?” remembering that a model only has to satisfy the defined need for the range of operation being considered.



- Here are simplifying assumptions that one might make to get a fundamental understanding of how the spring-mass systems behaves.
  - ⇒ The support to which the spring is attached is rigid, i.e., the support does not move as the mass attached to the spring moves. This assumption in effect says that the environment (everything outside of the system boundary) is independent of system motions.
  - ⇒ The spring is pure, i.e., it only has the characteristic (elasticity or springiness) for which it is named. A pure spring has negligible mass and energy dissipation (damping). This, of course, is an idealization as all springs have mass and dissipate energy upon cycling. If the spring mass is less than 10% of the mass attached to it, neglecting its mass is a reasonable assumption (except in high-speed applications). The energy dissipation in the spring is very small compared to other energy dissipation mechanisms in the system, so neglecting it is also reasonable.
  - ⇒ The spring is ideal, i.e., there is a linear relationship between spring force and spring displacement in the range of mass motion considered. This can be experimentally verified. The actual spring is a tension spring with some pretension (a force that pulls the coils of the spring together) and so there is a threshold force needed before the spring actually begins to stretch. Also the motion of the mass must be restricted to the range during which the spring is in tension, i.e., large amplitudes of motion of the mass are excluded from consideration.

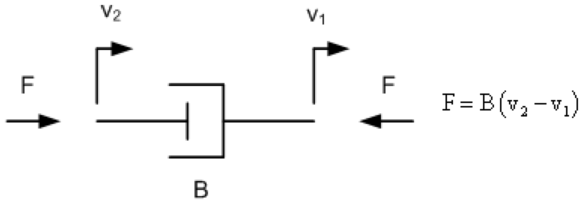
⇒ A schematic representation of a pure and ideal spring is shown, along with a graph of its force vs. displacement behavior.

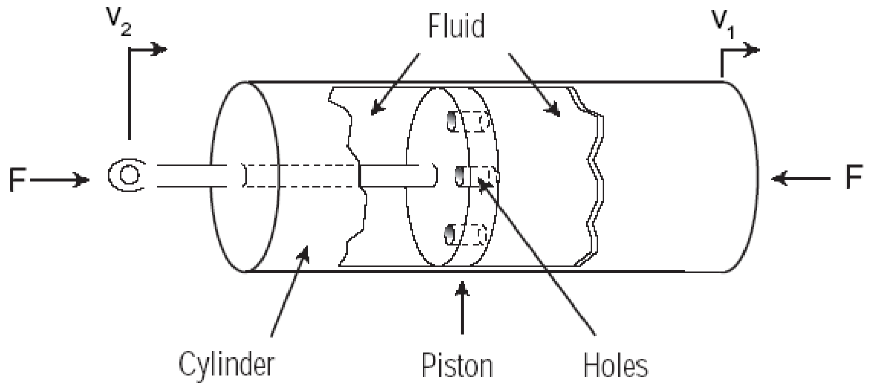


⇒ The attached mass can be treated as a rigid body, i.e., the attached mass does not deform in any way.

⇒ The mass moves with one degree of freedom in pure translation in the vertical plane. There is no out-of-plane motion and there is no rotational motion of the mass. The mass then can be treated as a point mass.

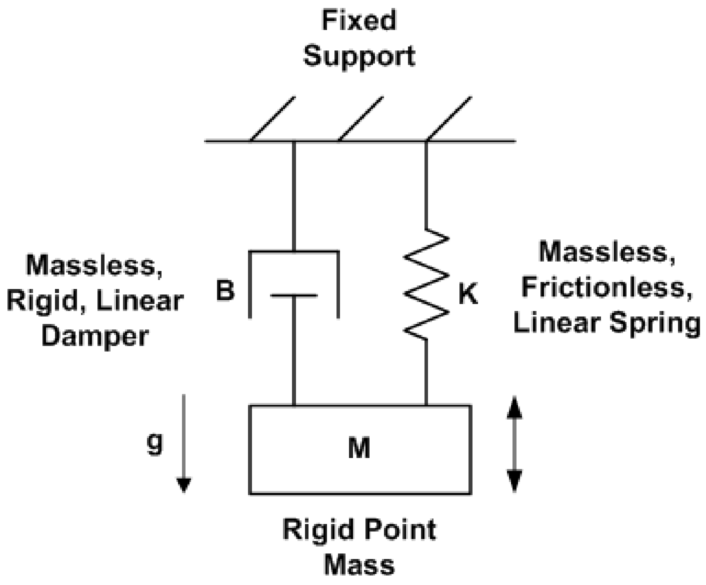
⇒ The friction in the system is not intentional; it is what we call parasitic, i.e., no energy dissipation mechanism has been intentionally designed into the system. Compared to the linear bearing friction, air damping due to the motion of the mass in the air is negligible, as is the energy dissipated by the spring. The friction in the linear bearing is the main source of energy dissipation and, based on engineering experience, is a combination of viscous fluid damping (proportional to the velocity of the mass and directed opposite to the mass motion) and dry-friction, or Coulomb damping (essentially constant in magnitude, independent of mass velocity, and directed opposite to the mass motion). Coulomb friction leads to a nonlinear mathematical model, while viscous fluid friction leads to a linear mathematical model. Linear equations are not only easier to solve, but are also easier to get insight into the physical behavior of the system from. However, the desire to have a linear mathematical model does not justify the assumption of viscous fluid damping and the omission of Coulomb damping. If this assumption is not based on sound engineering judgment, then the resulting mathematical model will not predict the actual behavior of the dynamic system. Let's assume that we can capture the major portion of the energy dissipation with the viscous fluid damping device called a damper. This device, like a spring, will be assumed to be pure and ideal, i.e., it has no mass and no springiness, only energy dissipation, and it behaves in a linear fashion, i.e., the force exerted at the ends of the damper, F, is proportional to the difference in velocity of the ends of the damper, $v_2 - v_1$, which is referred to as the relative velocity of the ends of the damper. The constant of proportionality is called the viscous damping coefficient, B. A shock absorber in your car is an example of a mechanical damper that is intentionally designed into every car. A schematic of a typical actual device along with its engineering symbol and linear relationship are shown.

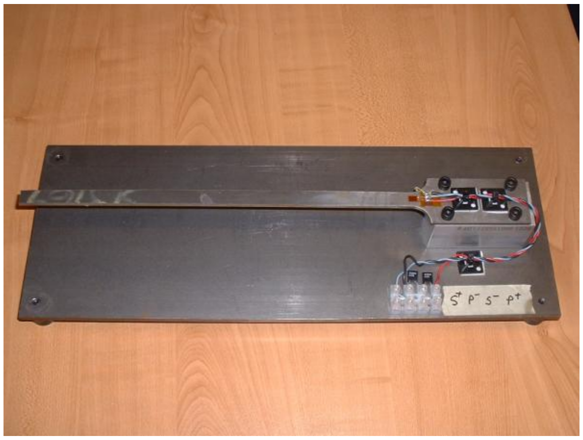




- ⇒ The system is vertical with the acceleration due to gravity, g, pointing downward and constant in value.
- ⇒ All parameters (mass, spring constant, viscous fluid damping coefficient) are constant, i.e., do not change with time or temperature, for example.

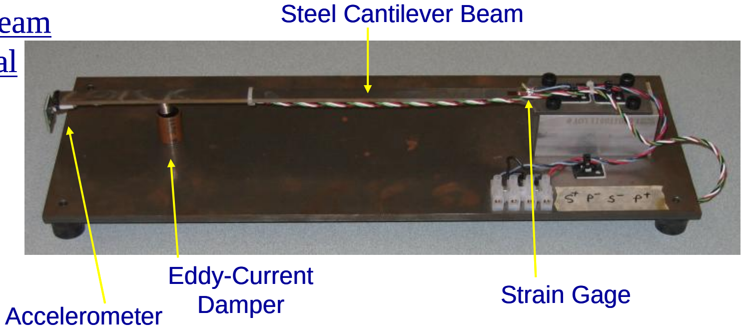
- With these assumptions, a physical model of this physical system looks like the schematic shown below.





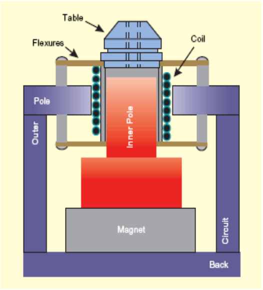
- Physical Modeling Example: As another example consider the simple cantilever beam made of steel. A cantilever beam is a beam fixed to ground at one end while the other end is free. The beam shown has strain gages attached to it at its base which can be used to determine the amount of beam deflection at the end of the beam when either the beam is loaded with a mass placed on the end of the beam or is simply plucked and allowed to vibrate. The beam bends when loaded, as it is flexible, not rigid – it has compliance or springiness and acts like a spring. The beam itself also has mass and when it is plucked and allowed to vibrate, the vibrations will eventually cease as there is dissipation or loss of energy during each cycle of its motion as the steel beam elastically deforms. So this system has the distributed characteristics of mass, springiness, and energy dissipation – the characteristics are distributed throughout the beam – you can't isolate any one of those characteristics. This is a very relevant dynamic mechanical system, as the diagram below shows. And end-tip accelerometer has been added to the cantilever beam, as has an eddy-current damper. The eddy-current damper consists of a copper pipe fixed to ground and a magnet attached to the beam. As the beam flexes, the magnet moves inside the copper pipe generating eddy currents in the pipe. These eddy currents generate a magnetic field that opposes the magnetic field of the moving magnet. The result is a damping force proportional to the velocity of the magnet. The cantilever beam is a key element in a computer hard-disk drive, a vibration exciter, and a MEMS (micro-electro-mechanical system) accelerometer. So understanding the behavior of a simple cantilever beam is a prerequisite to understanding the behavior of these complex systems.

Cantilever Beam Mechanical System

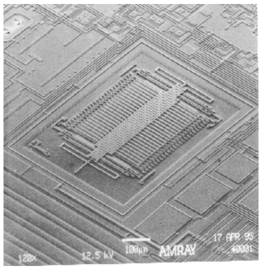


Hard-Drive Read-Write Head

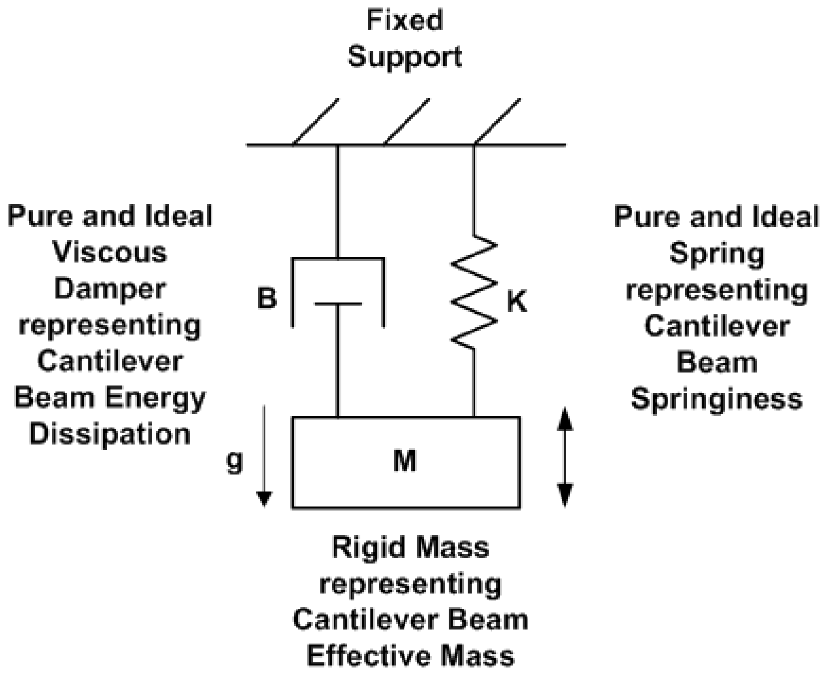
Vibration Exciter



MEMS Accelerometer



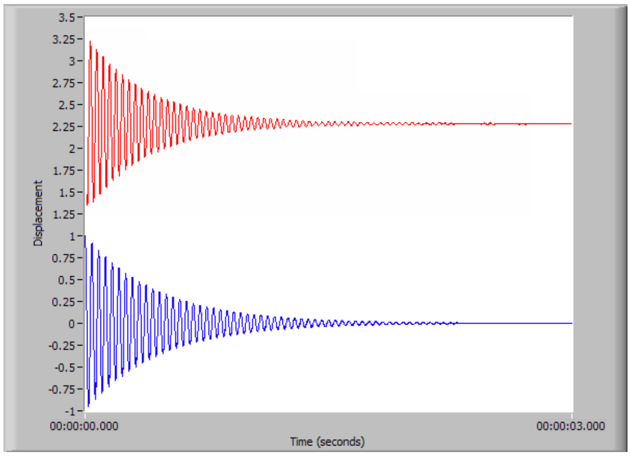
- What would a physical model of this cantilever-beam physical system look like? If we are concerned with motion of the tip of the beam, then a physical model of this physical system would look just like the spring-mass-damper system model shown below. While the structure of the model is correct, the challenge of quantifying the parameters in the physical model often makes physical modeling of real devices and systems difficult. This leads us to the next step in the process.



⇒ Model Parameter Identification

- The physical model has elements, not necessarily corresponding to actual physical components in the physical system, with characteristic parameters (spring constant, mass, resistance, inductance, thermal conductivity, fluid viscosity, thermal capacitance, etc.) whose numerical values must be identified. This is done either by numerical calculation, referencing standard handbooks, using vendor information, or through experiment.
- For the spring-mass system the parameters that were identified are: mass M (5.23 kg) of the attached block obtained by weighing the block; spring constant K (491 N/m) of the spring obtained by attaching two different masses to the spring and measuring the spring stretch for each mass. These two data points (weight and spring displacement) are then plotted so that the assumed linear (straight-line) behavior of the spring can be determined; and the viscous damping coefficient B (1.1 N-s/m) obtained by adjusting the value of B and curve fitting the predicted response with the measured response.
- For the cantilever beam physical model, the spring constant K in the model can be calculated from the dimensions of the beam (length L, width b, and thickness, h) and from the beam material property called the modulus of elasticity, E, but also can be determined by simply placing different weights on the end of the beam and measuring the end-point displacement. The values of the mass M and the viscous damping coefficient B are determined from the experimental plot of beam free oscillations assuming that the damped frequency of vibration is not much different from the undamped natural

frequency, an assumption valid for lightly-damped systems. The plot below shows the predicted vs. experimental time response when the beam is put into free vibration. The logarithmic decrement technique was used to determine the viscous damping coefficient.



$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

$$24 = \frac{1}{2\pi}\sqrt{\frac{42.84}{M}} \Rightarrow M = 0.00188 \text{ slugs}$$

$$\delta = \frac{1}{n}\ln\frac{B_1}{B_{n+1}} = \frac{1}{30}\ln\frac{1.036}{0.065} = 0.0923$$

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = 0.0147$$

$$\zeta = \frac{B}{2}\sqrt{\frac{1}{KM}} \Rightarrow B = 0.0083 \frac{\text{lbf}}{\text{ft/sec}}$$

⇒ Mathematical Model

- The laws of nature (physics, chemistry, biology) are applied to the physical model (not the physical system) and the mathematical equations describing the system are derived. Here is where the fundamental body of knowledge in science is applied in the process.
- The laws of physics are primarily used in our investigations, e.g., Newton's Laws, Conservation of Mass, Conservation of Energy, Kirchhoff's Voltage Law, Kirchhoff's Current Law, Faraday's Law, Ampere's Law, and Lenz's Law. We will use all these laws and apply them to physical models to generate mathematical models.
- In the case of the spring-mass system, we apply Newton's 2nd Law of Motion to a free-body diagram of the mass in our physical system model to obtain the equation of motion for the system, the mathematical model.
- In order to relate the physical model dynamic system performance to physical model hardware parameters, we use standard-form zero-, first-, and second-order dynamic system models with parameters steady-state gain, time constant, damping ratio, and natural frequency. The diagram shown explains this essential connection.

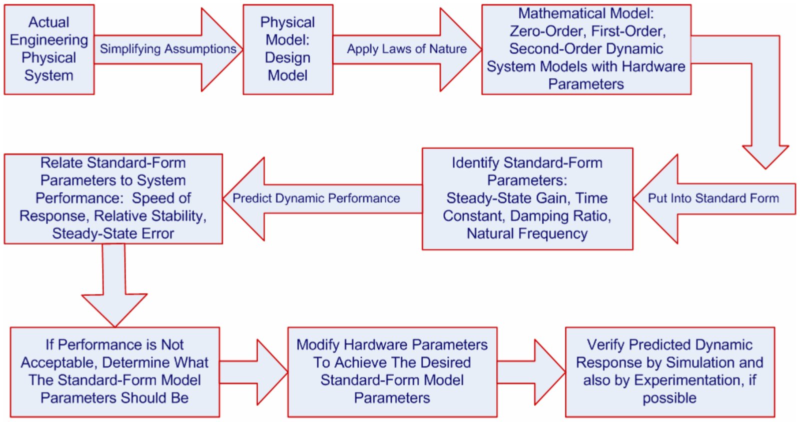


Diagram Showing How Physical Model Hardware Parameters Are Related to Physical Model Dynamic System Performance

⇒ Mathematical Analysis: Predicted Behavior

- The mathematical equations are solved either numerically by computer simulation or analytically (using appropriate theory to obtain a mathematical expression for the solution) to predict the behavior of the engineering system. The purpose of modeling is to gain insight into the behavior of the engineering system. Using a simpler model that allows for an analytical solution often leads to greater insight into system behavior than numerical solutions of a more complicated model.
- What should be the input into our system to make something happen, i.e., excite a response, so that we can compare a measured response with a predicted response? Engineers use specific inputs to evaluate performance of dynamic systems and compare alternative designs. The inputs used, both in the actual physical system and also in the mathematical model, lead to two complementary points of view: the time domain and the frequency domain. Understanding both points of view is essential for an engineer. Together, time domain and frequency domain give a complete picture of the behavior of a dynamic system. They are essential and complementary.
- This step is only half the story, for computer simulation or mathematical analysis without experimental verification is at best questionable, and at worst, useless.

⇒ System Measurement

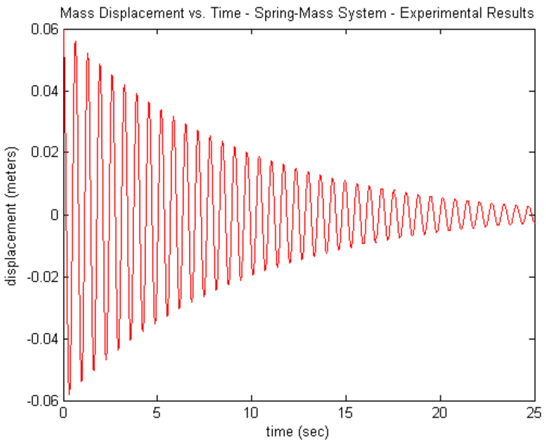
- Experiments are performed on the engineering system to validate the predicted system response. Both time-response and frequency-response measurements are made on the actual system or device. If the physical system is a concept in the design process, then experimental verification is accomplished by selective experimentation focusing on questionable modeling elements.

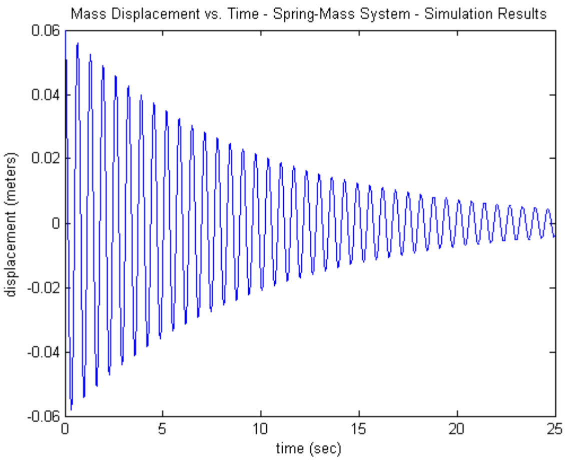
⇒ Measurement Analysis

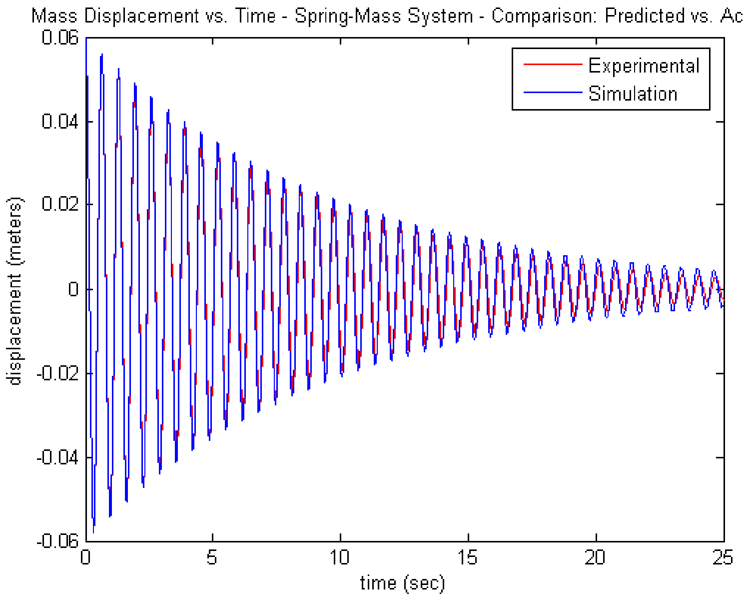
- The experimental data, both time-domain and frequency-domain, must be reviewed and analyzed for accuracy.

⇒ Comparison: Mathematical Predications vs. Experimental Observations

- If the model predictions compare favorably to the experimental observations, then the model is adequate. If not, the experimental measurements, the analytical predictions, and the physical-model simplifying assumptions must be reviewed for accuracy. The physical model might need to be modified to capture the system characteristics which are important and were not initially included.
- Parasitic, or secondary, effects (e.g., saturation, nonlinear effects, time delays, hysteresis, Coulomb friction, and gear backlash) are added to the physical model to determine if each effect is significant, or if cumulatively they have adverse effects.
- Eventually, a truth model, which is as realistic a model one could develop, is used to validate system performance prior to hardware implementation. This often eliminates the need for hardware prototyping. The advantages over the build-and-test approach are staggering.
- Let’s view below the comparison of the predicted response with the experimental response for the spring-mass system and the draw some conclusions.







- The agreement between the predicted response and the actual measured response is quite good. The frequency of the oscillations, obtained by counting the number of cycles of the sine wave in a specified amount of time (here there about 15 cycles in 10 seconds or 1.5 cycles/sec) is the same and the oscillations occur about the zero position which corresponds to the static equilibrium position. The difference in amplitude between the two responses becomes more noticeable as the oscillations diminish. This is as expected because, from engineering experience, Coulomb friction will begin to dominate the response over viscous fluid friction when the system begins to slow down, and the physical model does not contain a Coulomb-friction term.

⇒ Design Changes

- If the model is adequate, but performance is inadequate, then design changes (e.g., change actuator or its location, add a control system) are in order, and the whole engineering system investigation process then starts over again for the revised system.