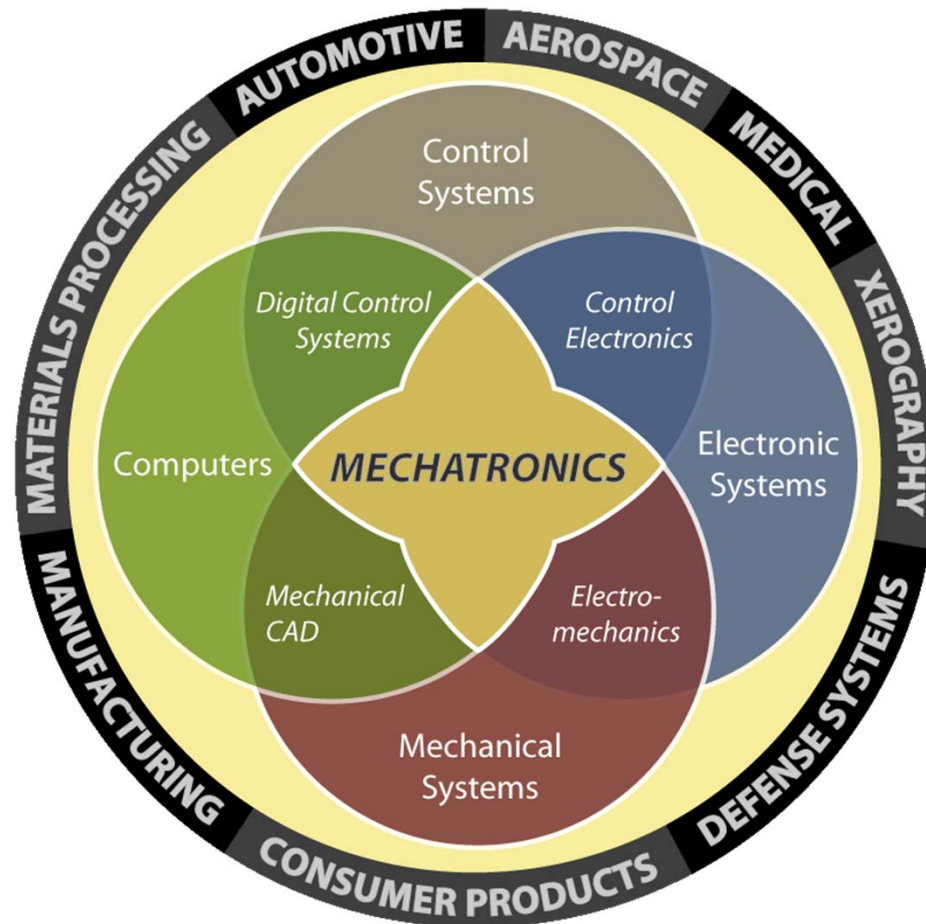
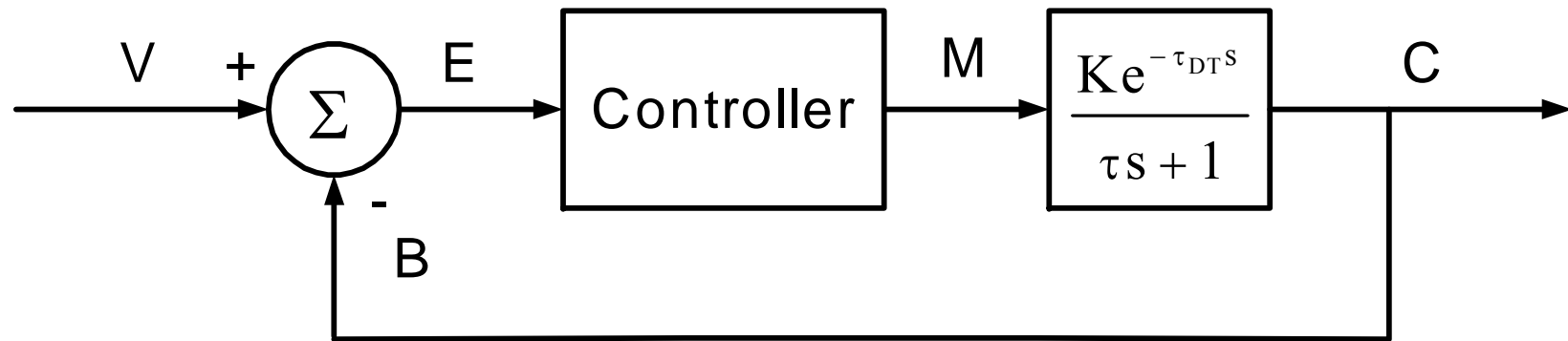


Control of a First-Order Process with Dead Time



Control of a First-Order Process with Dead Time



The most commonly used model to describe the dynamics of chemical processes is the First-Order Plus Time Delay Model. By proper choice of τ_{DT} and τ , this model can be made to represent the dynamics of many industrial processes.

- Time delays or dead-times (DT's) between inputs and outputs are very common in industrial processes, engineering systems, economical, and biological systems.
- Transportation and measurement lags, analysis times, computation and communication lags all introduce DT's into control loops.
- DT's are also used to compensate for model reduction where high-order systems are represented by low-order models with delays.
- Two major consequences:
 - Complicates the analysis and design of feedback control systems
 - Makes satisfactory control more difficult to achieve

- Any delay in measuring, in controller action, in actuator operation, in computer computation, and the like, is called *transport delay* or *dead time*, and it always reduces the stability of a system and limits the achievable response time of the system.



$q_i(t)$ = input to dead-time element

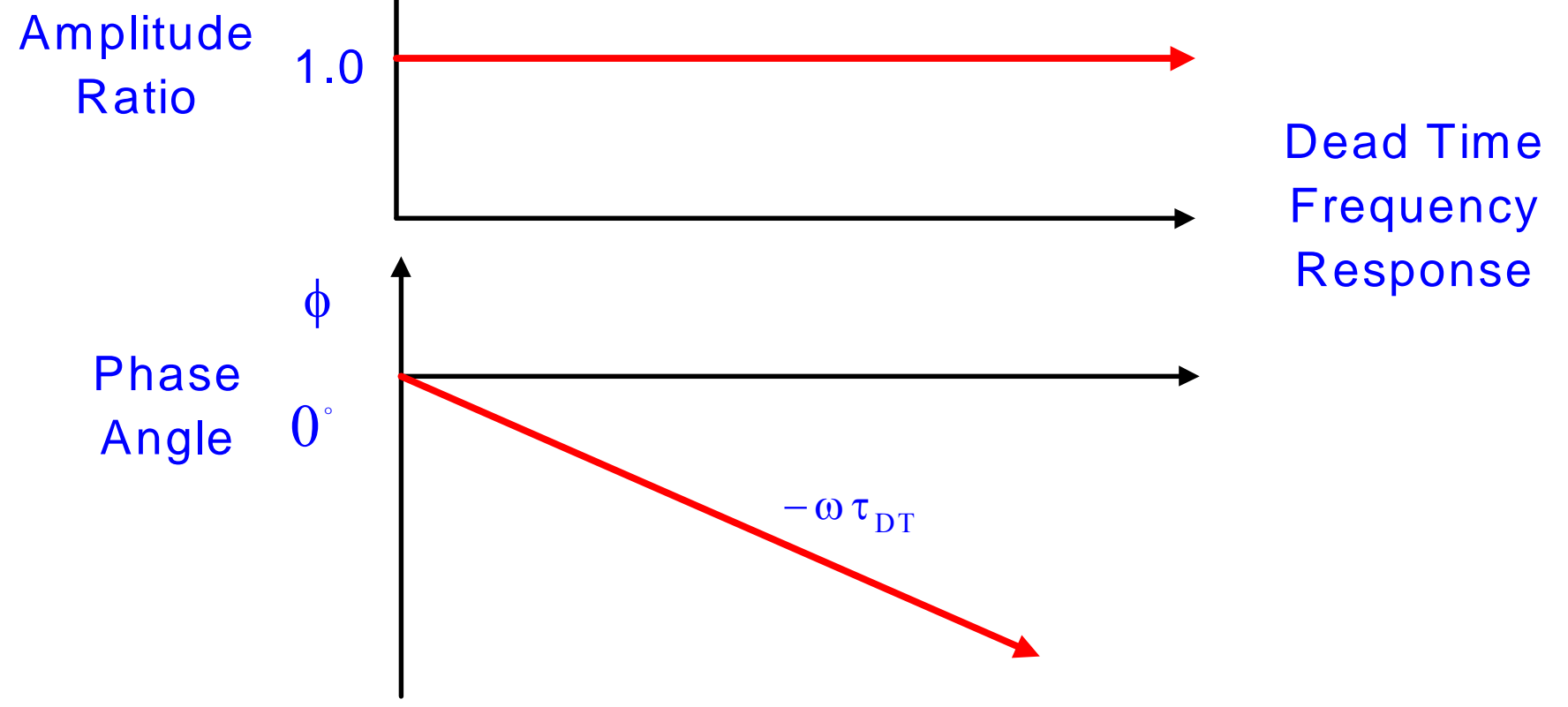
$q_o(t)$ = output of dead-time element = $q_i(t - \tau_{DT})u(t - \tau_{DT})$

$u(t - \tau_{DT}) = 1$ for $t \geq \tau_{DT}$

$u(t - \tau_{DT}) = 0$ for $t < \tau_{DT}$

Laplace Transform

$$L[f(t - \tau_{DT})u(t - \tau_{DT})] = e^{-\tau_{DT}s}F(s)$$



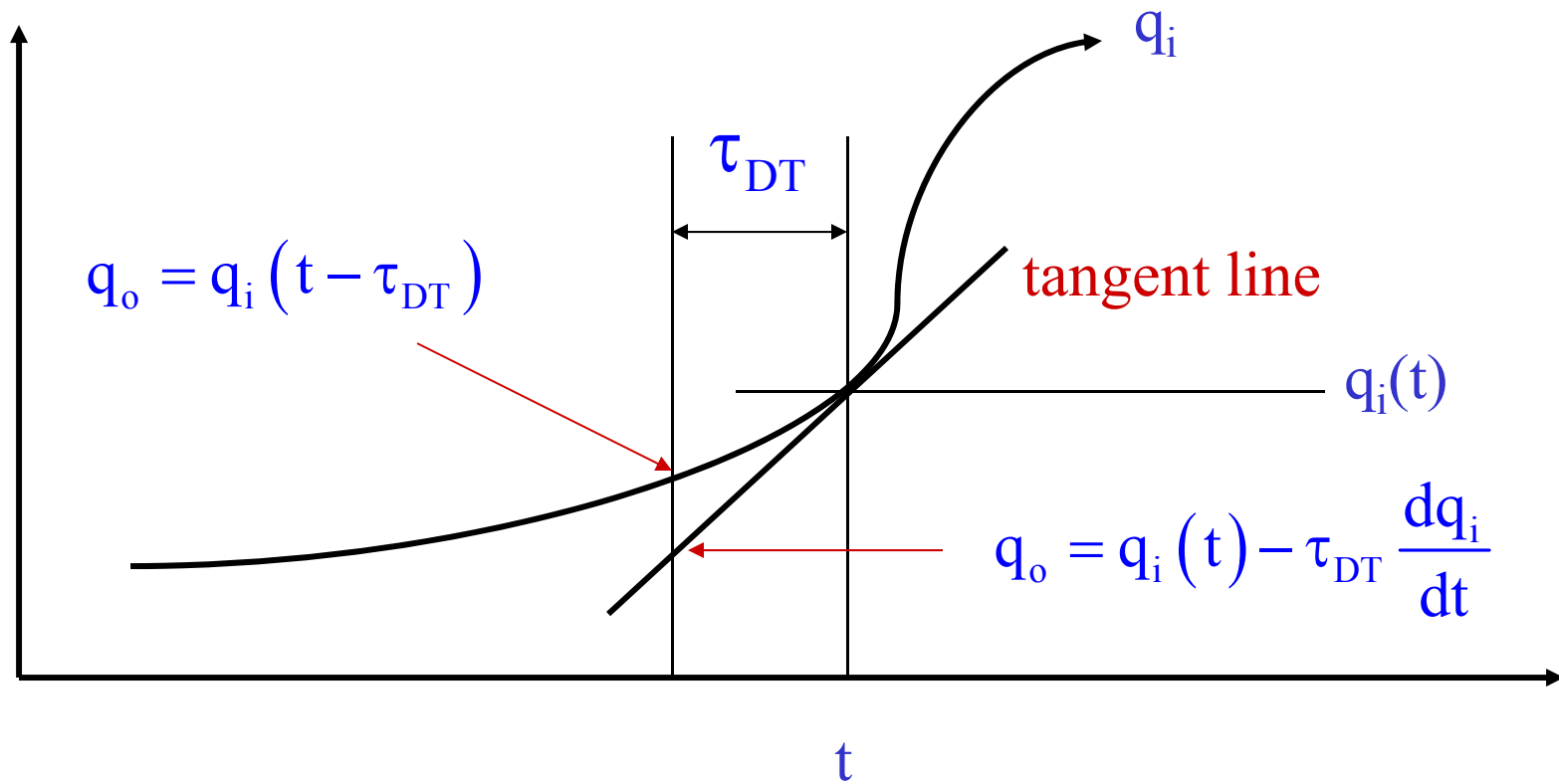
- Dead-Time Approximations

- The simplest dead-time approximation can be obtained graphically or by taking the first two terms of the Taylor series expansion of the Laplace transfer function of a dead-time element, τ_{DT} .

$$\frac{Q_o}{Q_i}(s) = e^{-\tau_{DT}s} \approx 1 - \tau_{DT}s \quad q_o(t) \approx q_i(t) - \tau_{DT} \frac{dq_i}{dt}$$

- The accuracy of this approximation depends on the dead time being sufficiently small relative to the rate of change of the slope of $q_i(t)$. If $q_i(t)$ were a ramp (constant slope), the approximation would be perfect for any value of τ_{DT} . When the slope of $q_i(t)$ varies rapidly, only small τ_{DT} 's will give a good approximation.
- A frequency-response viewpoint gives a more general accuracy criterion; if the amplitude ratio and the phase of the approximation are sufficiently close to the exact frequency response curves of for the range of frequencies present in $q_i(t)$, then the approximation is valid.

Dead-Time Graphical Approximation



- The Pade approximants provide a family of approximations of increasing accuracy (and complexity):

$$e^{-\tau s} = \frac{e^{-\frac{\tau s}{2}}}{e^{\frac{\tau s}{2}}} \approx \frac{1 - \frac{\tau s}{2} + \frac{\tau^2 s^2}{8} + \dots + \frac{\left(-\frac{\tau s}{2}\right)^k}{k!}}{1 + \frac{\tau s}{2} + \frac{\tau^2 s^2}{8} + \dots + \frac{\left(\frac{\tau s}{2}\right)^k}{k!}}$$

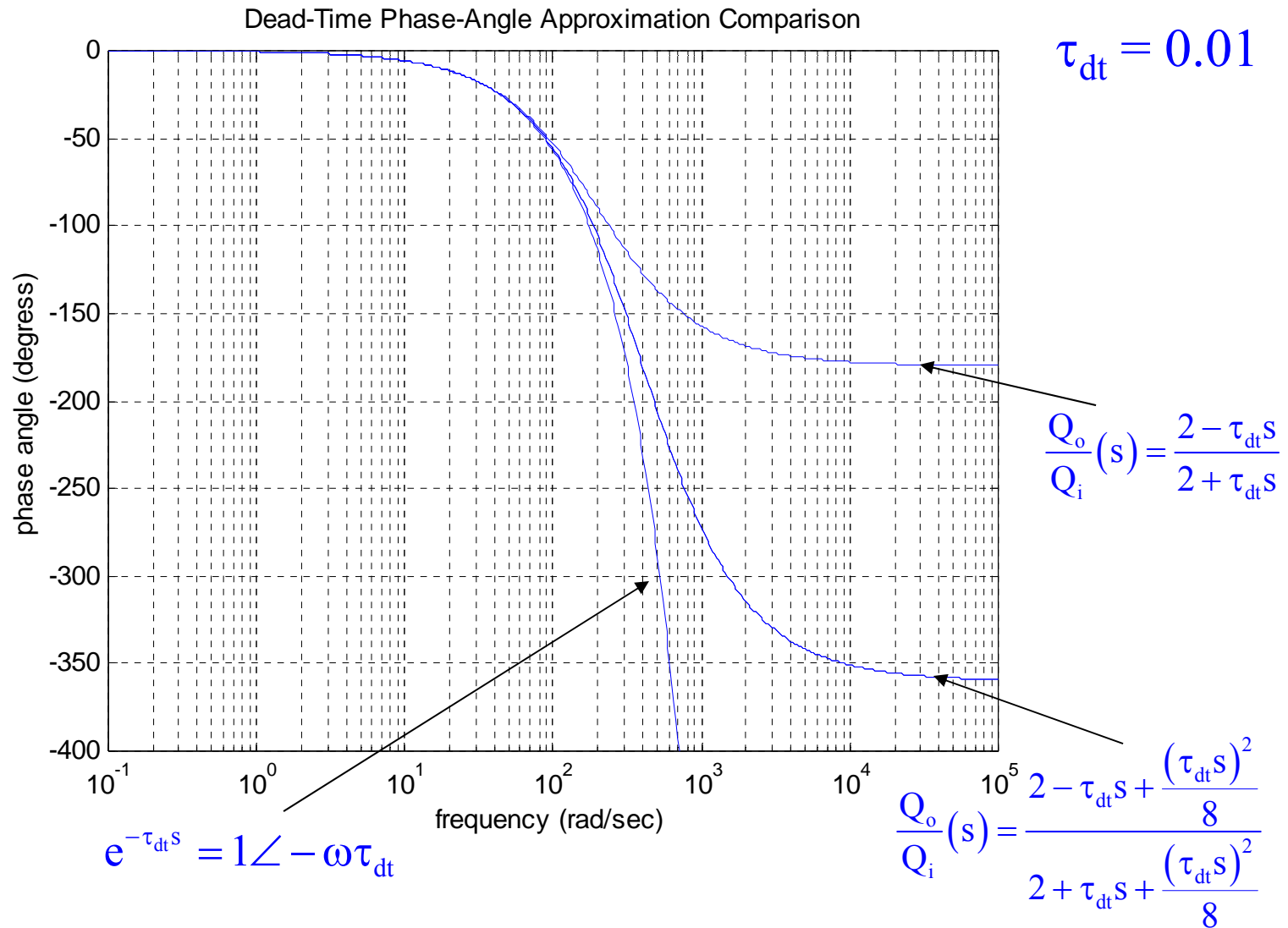
- In some cases, a very crude approximation given by a first-order lag is acceptable:

$$\frac{Q_o}{Q_i}(s) = e^{-\tau_{DT} s} \approx \frac{1}{\tau_{DT} s + 1}$$

- Pade Approximation:
 - Transfer function is all pass, i.e., the magnitude of the transfer function is 1 for all frequencies.
 - Transfer function is non-minimum phase, i.e., it has zeros in the right-half plane.
 - As the order of the approximation is increased, it approximates the low-frequency phase characteristic with increasing accuracy.
- Another approximation with the same properties:

$$e^{-\tau s} = \frac{e^{-\frac{\tau s}{2}}}{e^{\frac{\tau s}{2}}} \approx \frac{\left(1 - \frac{\tau s}{2k}\right)^k}{\left(1 + \frac{\tau s}{2k}\right)^k}$$

Dead-time Approximation Comparison

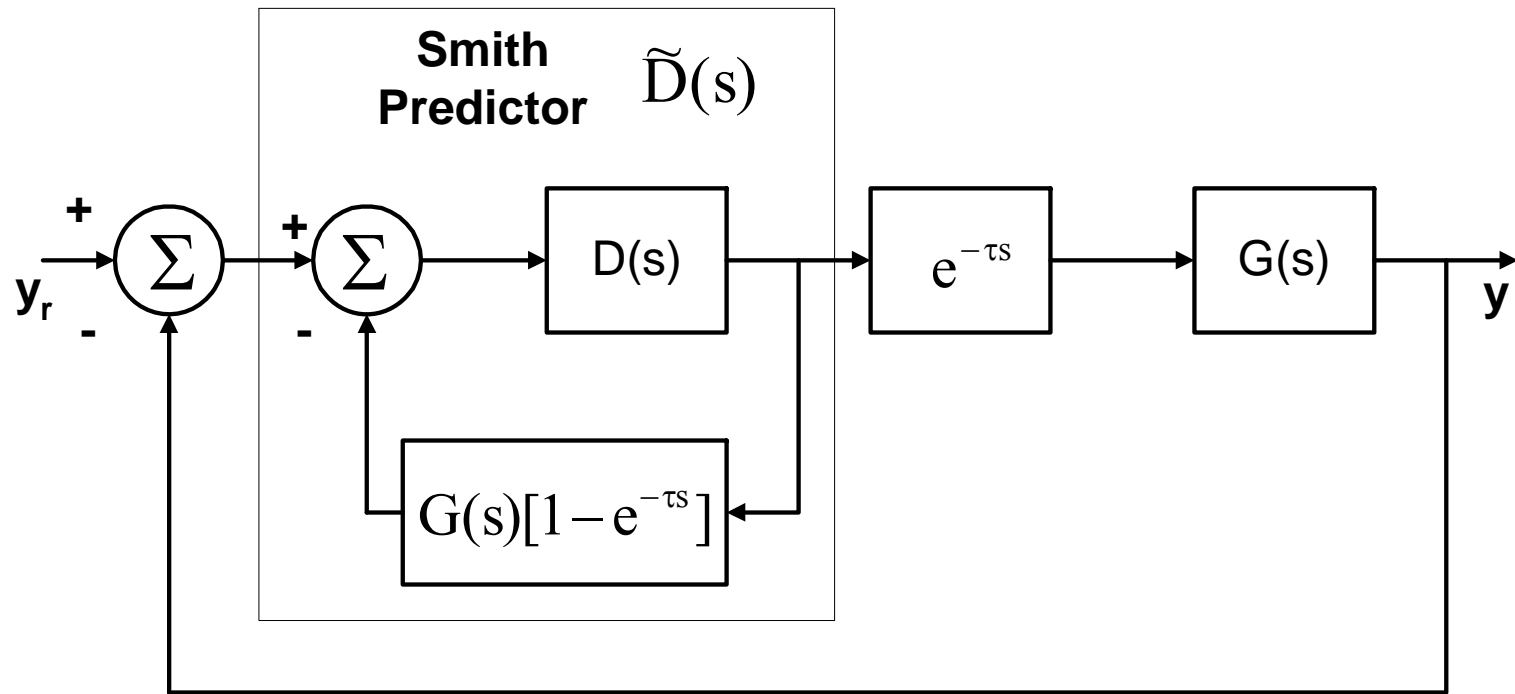


- Observations:

- Instability in feedback control systems results from an imbalance between system dynamic lags and the strength of the corrective action.
- When DT's are present in the control loop, controller gains have to be reduced to maintain stability.
- The larger the DT is relative to the time scale of the dynamics of the process, the larger the reduction required.
- The result is poor performance and sluggish responses.
- Unbounded negative phase angle aggravates stability problems in feedback systems with DT's.

- The time delay increases the phase shift proportional to frequency, with the proportionality constant being equal to the time delay.
- The amplitude characteristic of the Bode plot is unaffected by a time delay.
- Time delay always decreases the phase margin of a system.
- Gain crossover frequency is unaffected by a time delay.
- Frequency-response methods treat dead times exactly.
- Differential equation methods require an approximation for the dead time.
- To avoid compromising performance of the closed-loop system, one must account for the time delay explicitly, e.g., Smith Predictor.

Smith Predictor



$$\tilde{D}(s) = \frac{D(s)}{1 + (1 - e^{-\tau s})D(s)G(s)} \quad \frac{y}{y_r} = \frac{\tilde{D}(s)G(s)e^{-\tau s}}{1 + \tilde{D}(s)G(s)e^{-\tau s}} = \frac{D(s)G(s)}{1 + D(s)G(s)} e^{-\tau s}$$

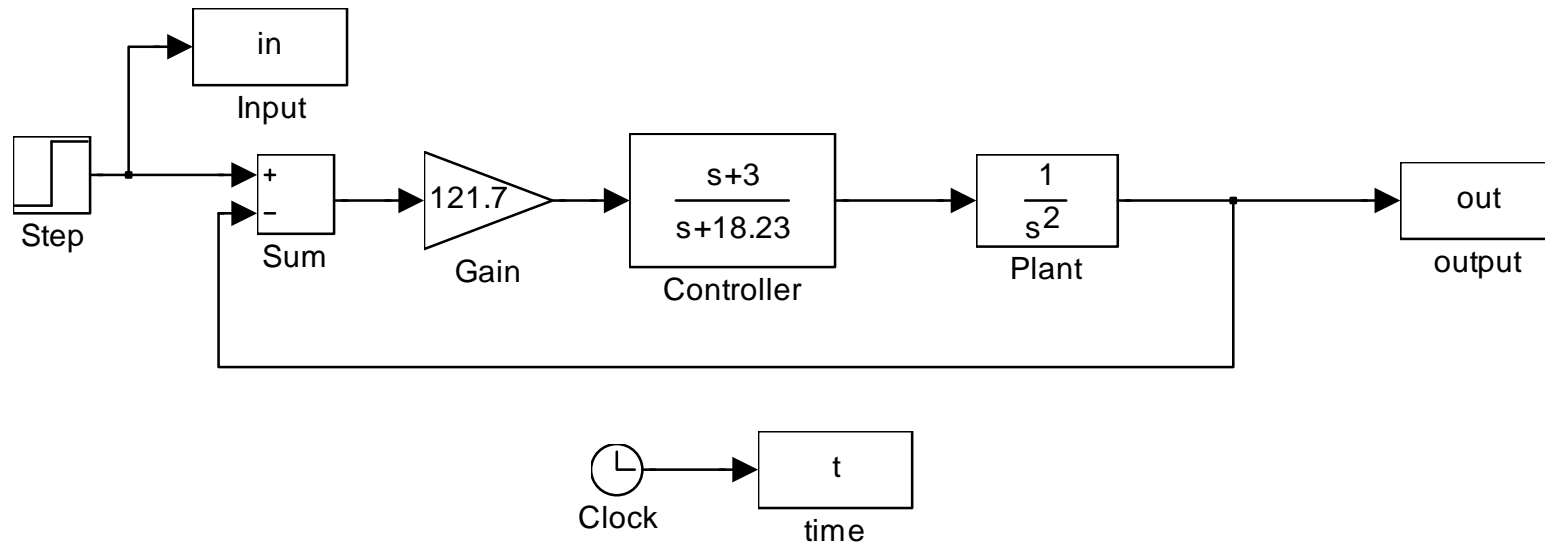
- $D(s)$ is a suitable compensator for a plant whose transfer function, in the absence of time delay, is $G(s)$.
- With the compensator that uses the Smith Predictor, the closed-loop transfer function, except for the factor $e^{-\tau s}$, is the same as the transfer function of the closed-loop system for the plant without the time delay and with the compensator $D(s)$.
- The time response of the closed-loop system with a compensator that uses a Smith Predictor will thus have the same shape as the response of the closed-loop system without the time delay compensated by $D(s)$; the only difference is that the output will be delayed by τ seconds.

- Implementation Issues

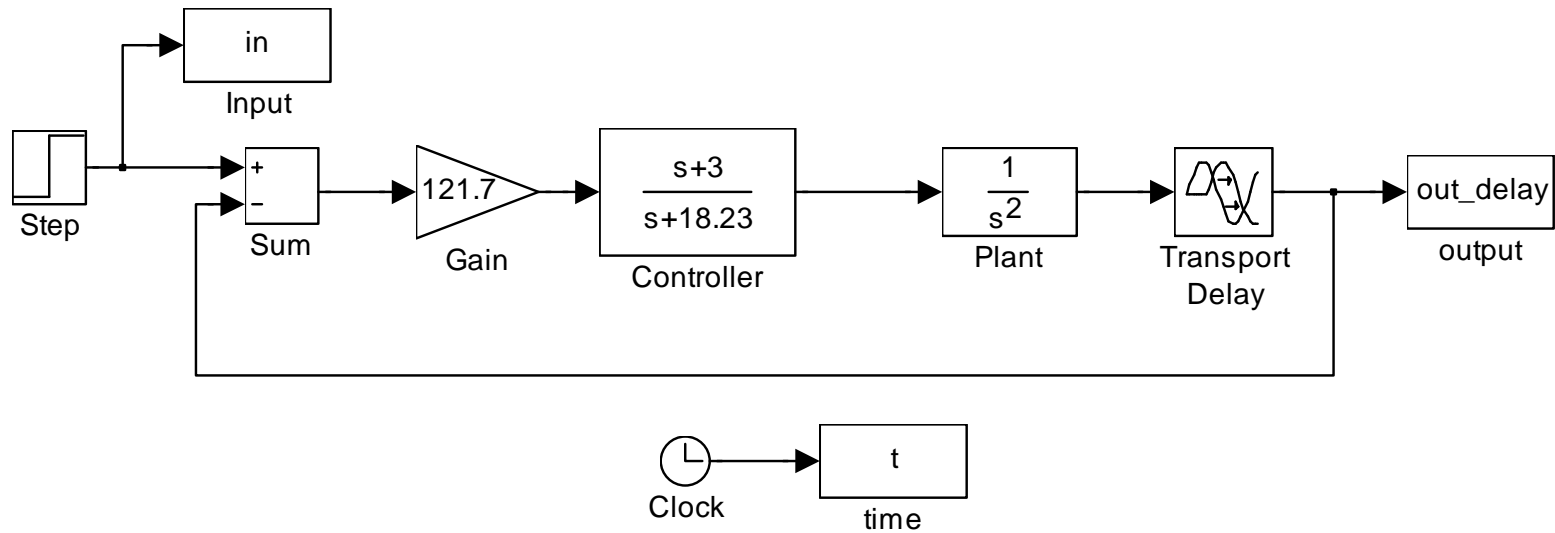
- You must know the plant transfer function and the time delay with reasonable accuracy.
- You need a method of realizing the pure time delay that appears in the feedback loop, e.g., Pade approximation:

$$e^{-\tau s} = \frac{e^{-\frac{\tau s}{2}}}{e^{\frac{\tau s}{2}}} \approx \frac{1 - \frac{\tau s}{2} + \frac{\tau^2 s^2}{8} + \dots + \frac{\left(-\frac{\tau s}{2}\right)^k}{k!}}{1 + \frac{\tau s}{2} + \frac{\tau^2 s^2}{8} + \dots + \frac{\left(\frac{\tau s}{2}\right)^k}{k!}}$$

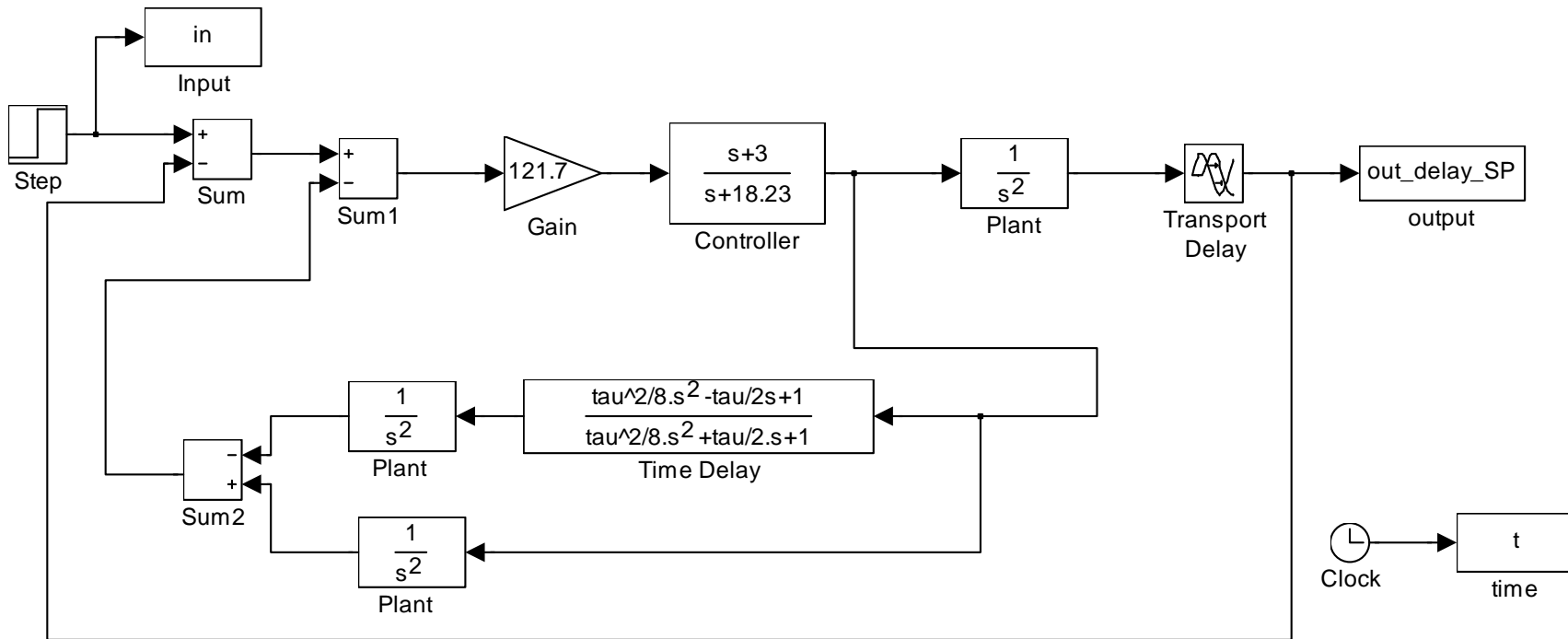
Example Problem



Basic Feedback Control System with Lead Compensator

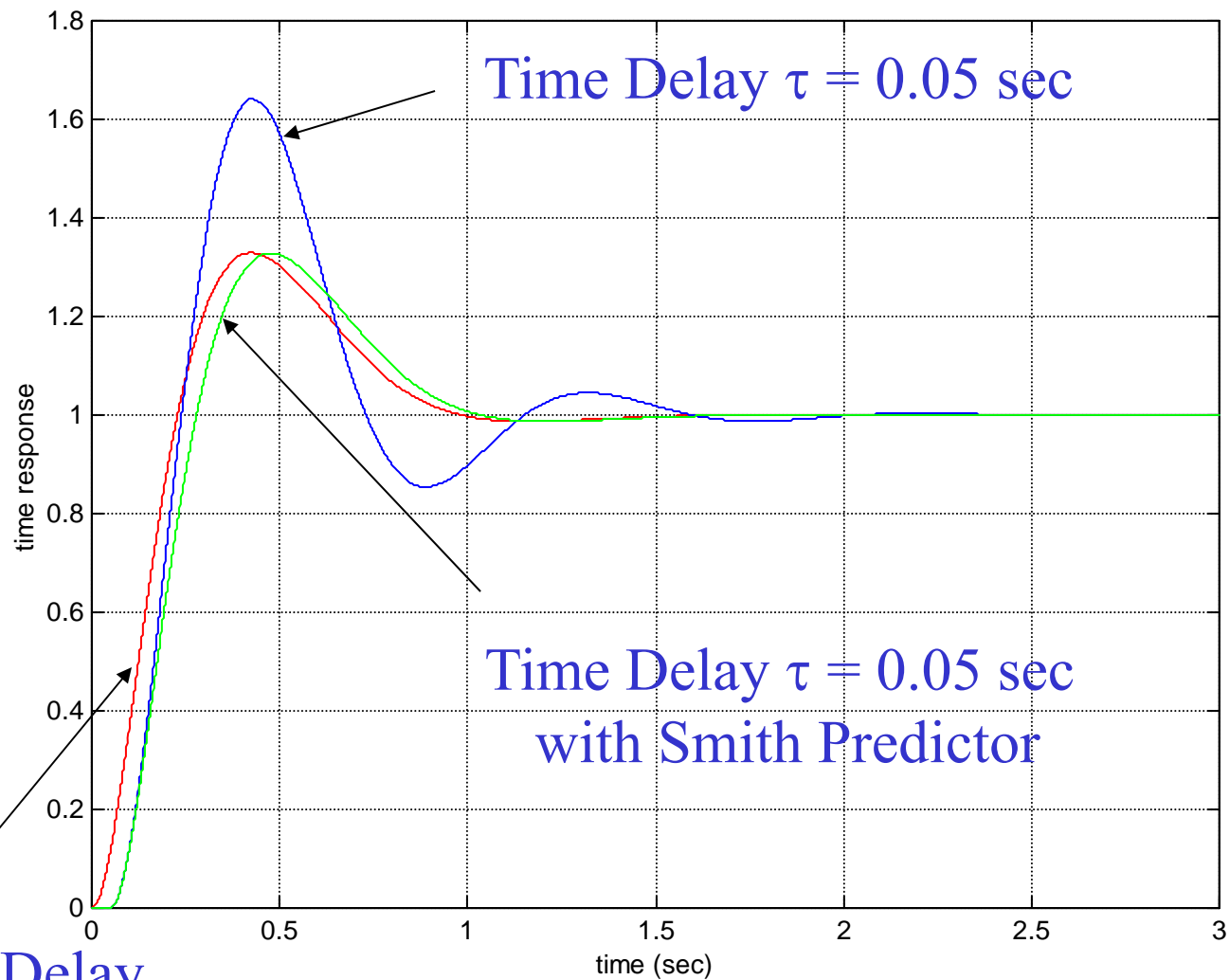


Basic Feedback Control System with Lead Compensator
 BUT with Time Delay $\tau = 0.05$ sec



**Basic Feedback Control System with Lead Compensator
 BUT with Time Delay $\tau = 0.05$ sec
 AND Smith Predictor**

System Step Responses



No Time Delay

- Comments

- The system with the Smith Predictor tracks reference variations with a time delay.
- The Smith Predictor minimizes the effect of the DT on stability as model mismatching is bound to exist. This however still allows tighter control to be used.
- What is the effect of a disturbance? If the disturbances are measurable, the regulation capabilities of the Smith Predictor can be improved by the addition of a feedforward controller.

- Minimum-Phase and Nonminimum-Phase Systems
 - Transfer functions having *neither* poles nor zeros in the RHP are *minimum-phase* transfer functions.
 - Transfer functions having *either* poles or zeros in the RHP are *nonminimum-phase* transfer functions.
 - For systems with the same magnitude characteristic, the range in phase angle of the minimum-phase transfer function is minimum among all such systems, while the range in phase angle of any nonminimum-phase transfer function is greater than this minimum.
 - For a minimum-phase system, the transfer function can be uniquely determined from the magnitude curve alone. For a nonminimum-phase system, this is not the case.

Consider as an example the following two systems:

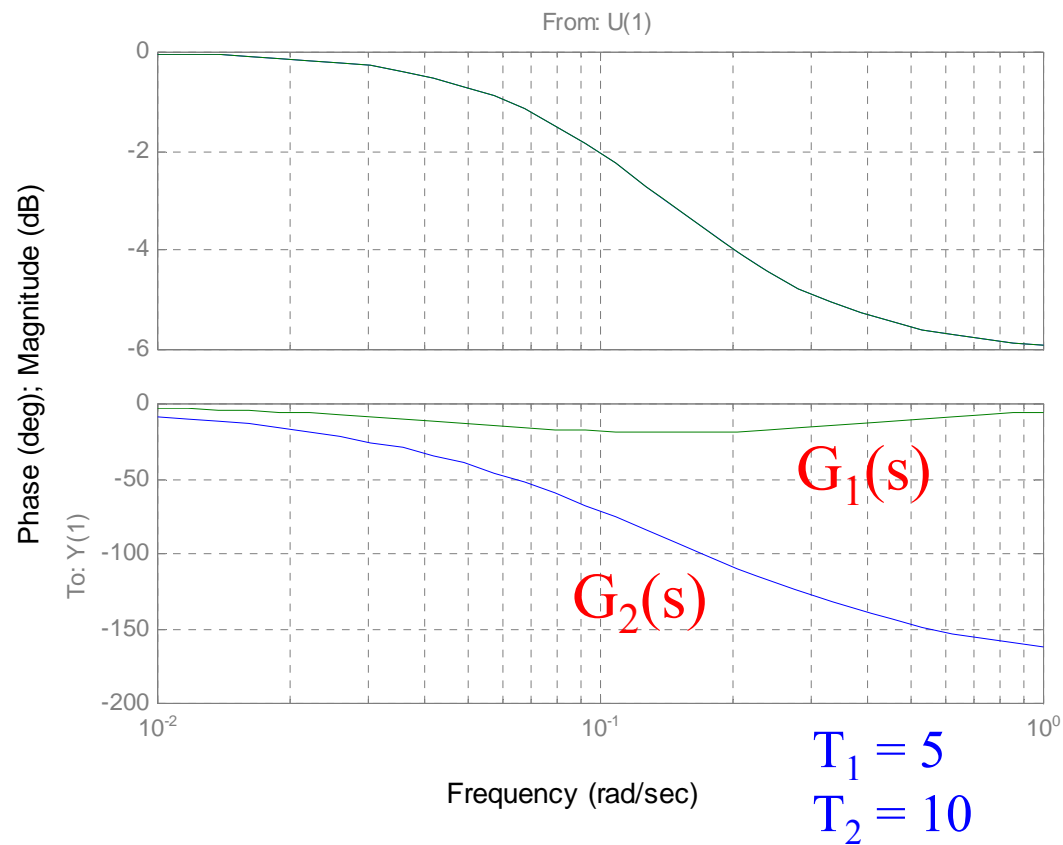
$$G_1(s) = \frac{1 + T_1 s}{1 + T_2 s}$$

$$G_2(s) = \frac{1 - T_1 s}{1 + T_2 s}$$

$$0 < T_1 < T_2$$

A small amount of change in magnitude produces a small amount of change in the phase of $G_1(s)$ but a much larger change in the phase of $G_2(s)$.

Bode Diagrams



- These two systems have the same magnitude characteristics, but they have different phase-angle characteristics.
- The two systems differ from each other by the factor:

$$G(s) = \frac{1 - T_1 s}{1 + T_1 s}$$

- This factor has a magnitude of unity and a phase angle that varies from 0° to -180° as ω is increased from 0 to ∞ .
- For the stable minimum-phase system, the magnitude and phase-angle characteristics are uniquely related. This means that if the magnitude curve is specified over the entire frequency range from zero to infinity, then the phase-angle curve is uniquely determined, and vice versa. This is called **Bode's Gain-Phase** relationship.

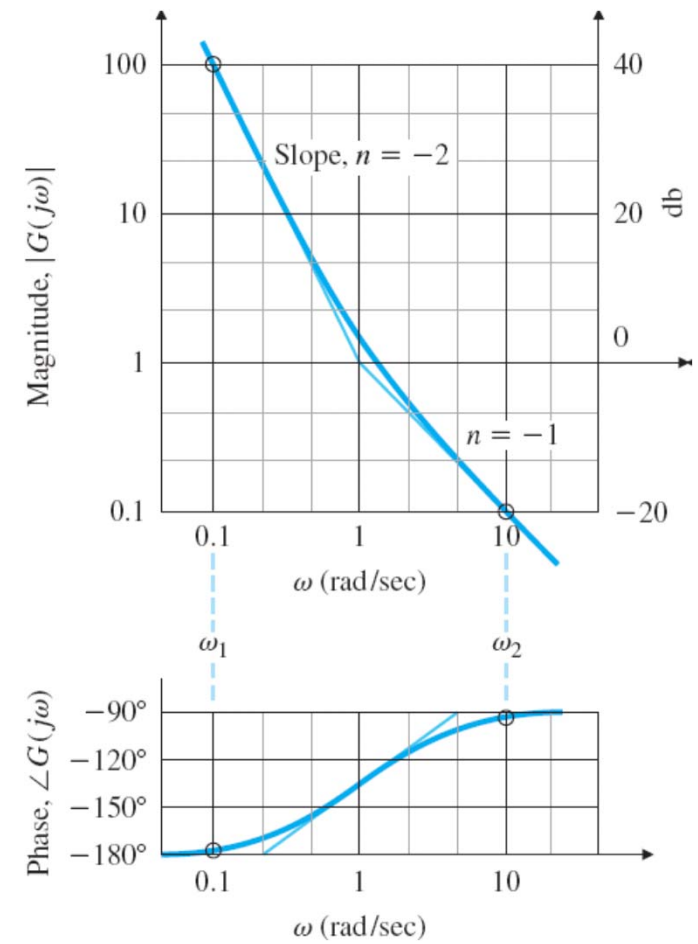
- Bode's Gain-Phase Relationship

- For any minimum-phase system (i.e., one with no RHP zeros or poles), the phase of $G(j\omega)$ is uniquely related to the magnitude of $G(j\omega)$.

When the slope of the magnitude vs. ω on a log-log scale persists at a constant value for approximately a decade of frequency, the relationship is particularly simple and is given by the relationship

$$\angle G(j\omega) \cong n \times 90^\circ$$

where n is the slope of the magnitude curve in units of decade of amplitude per decade of frequency.

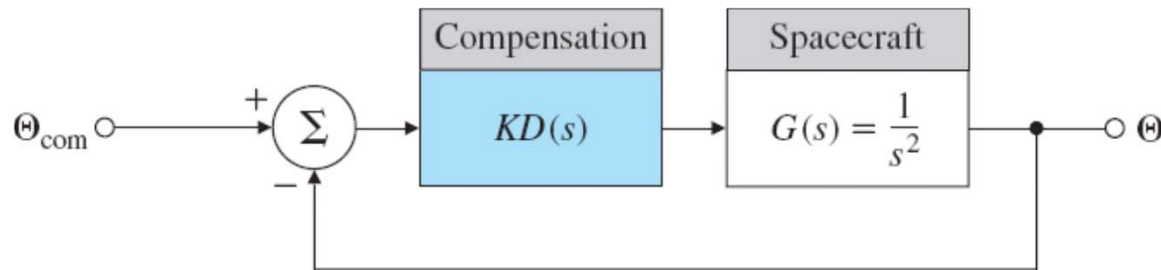


- For stability we want the angle of $G(j\omega) > -180^\circ$ for a $PM > 0$. Therefore, we adjust the magnitude curve so that it has a slope of -1 at the crossover frequency, ω_c , that is, where the magnitude = 1.
- If the slope is -1 for a decade above and below the crossover frequency, then the $PM \approx 90^\circ$.
- However, to ensure a reasonable PM, it is usually necessary only to insist that a slope of -1 (-20 dB per decade) persist for a decade in frequency that is centered at the crossover frequency.
- So a design procedure is to adjust the slope of the magnitude curve so that it crosses over magnitude 1 with a slope of -1 for a decade around ω_c to provide acceptable PM, and hence adequate damping. Then adjust the system gain to give a ω_c that will yield the desired bandwidth (and, hence, speed of response).

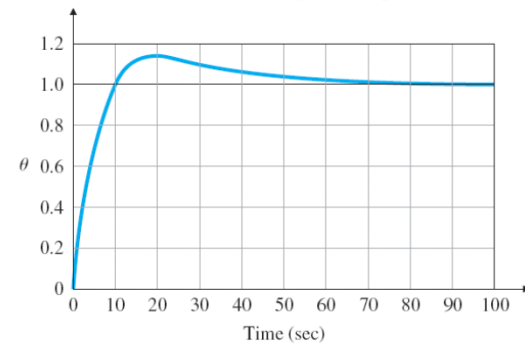
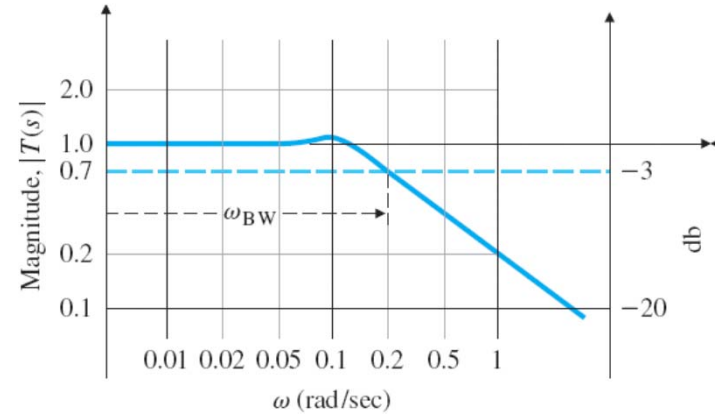
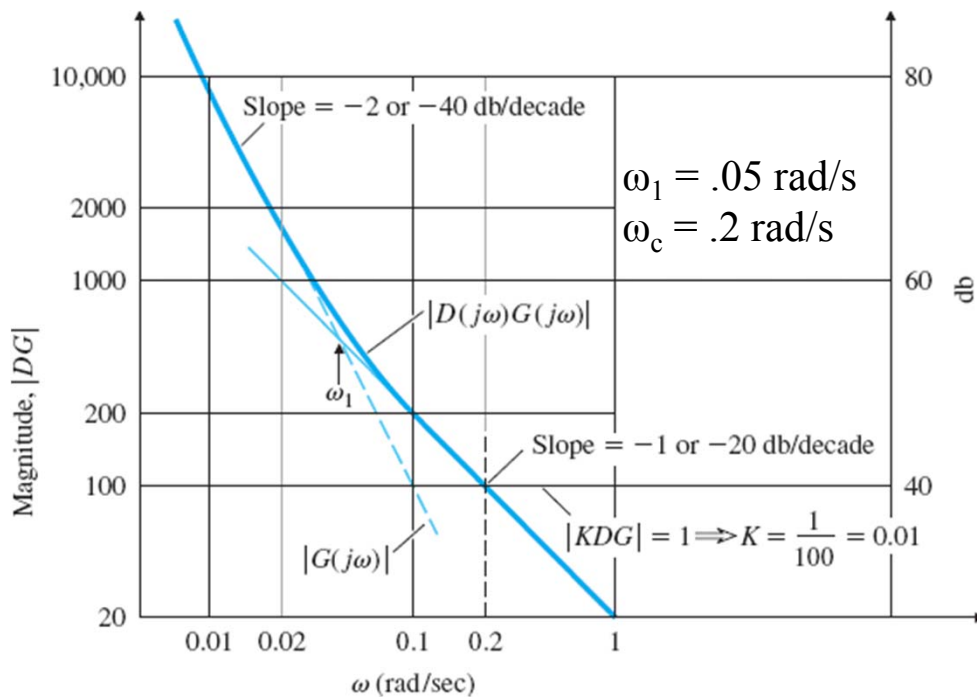
Simple Design Example

Design Objective: good damping and an approximate bandwidth of 0.2 rad/s.

$$KD(s) = K(T_D s + 1) = 0.01(20s + 1)$$



Adjust gain K to produce the desired bandwidth and adjust the breakpoint $\omega_1 = 1/T_D$ to provide the -1 slope at ω_c .

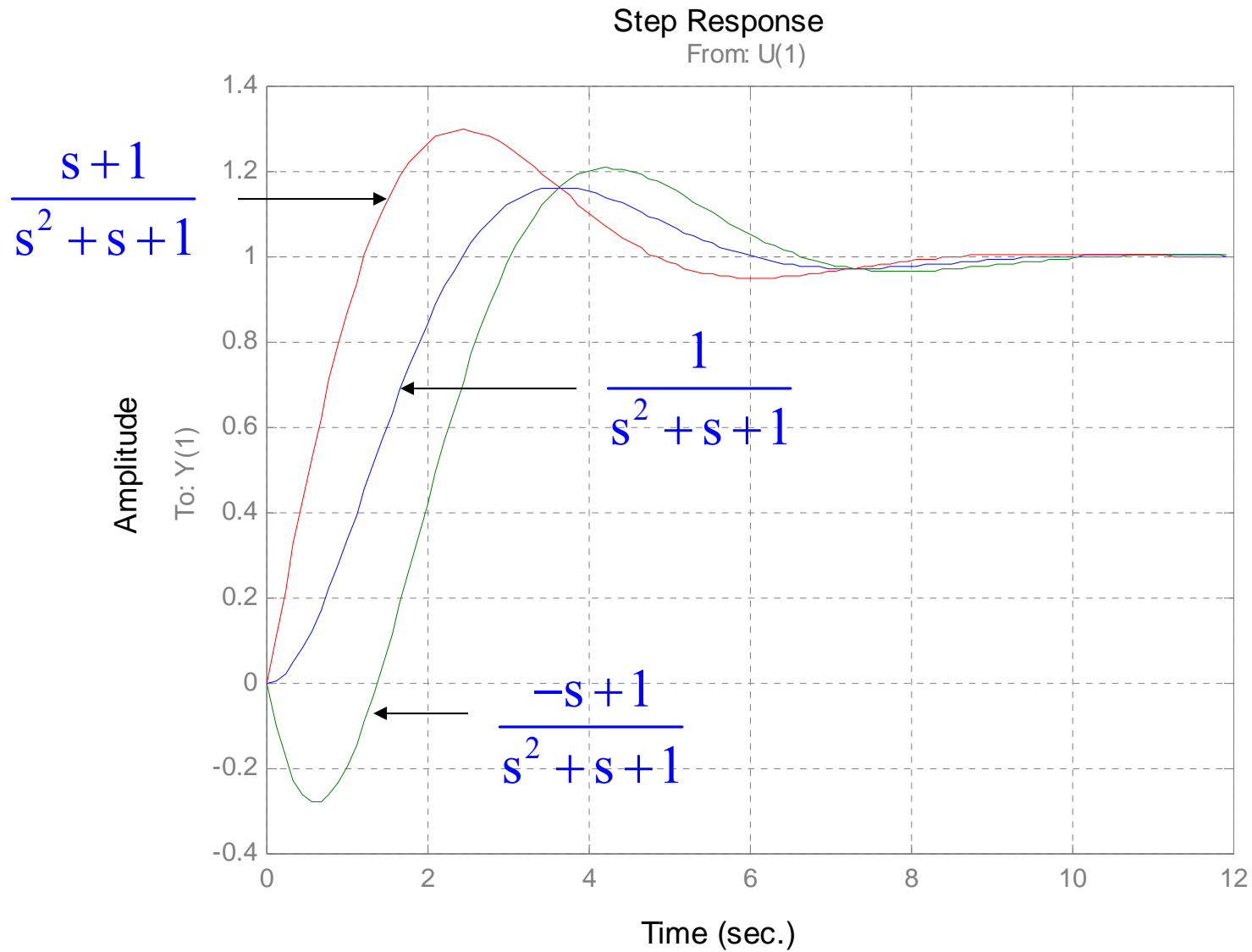


Step Response

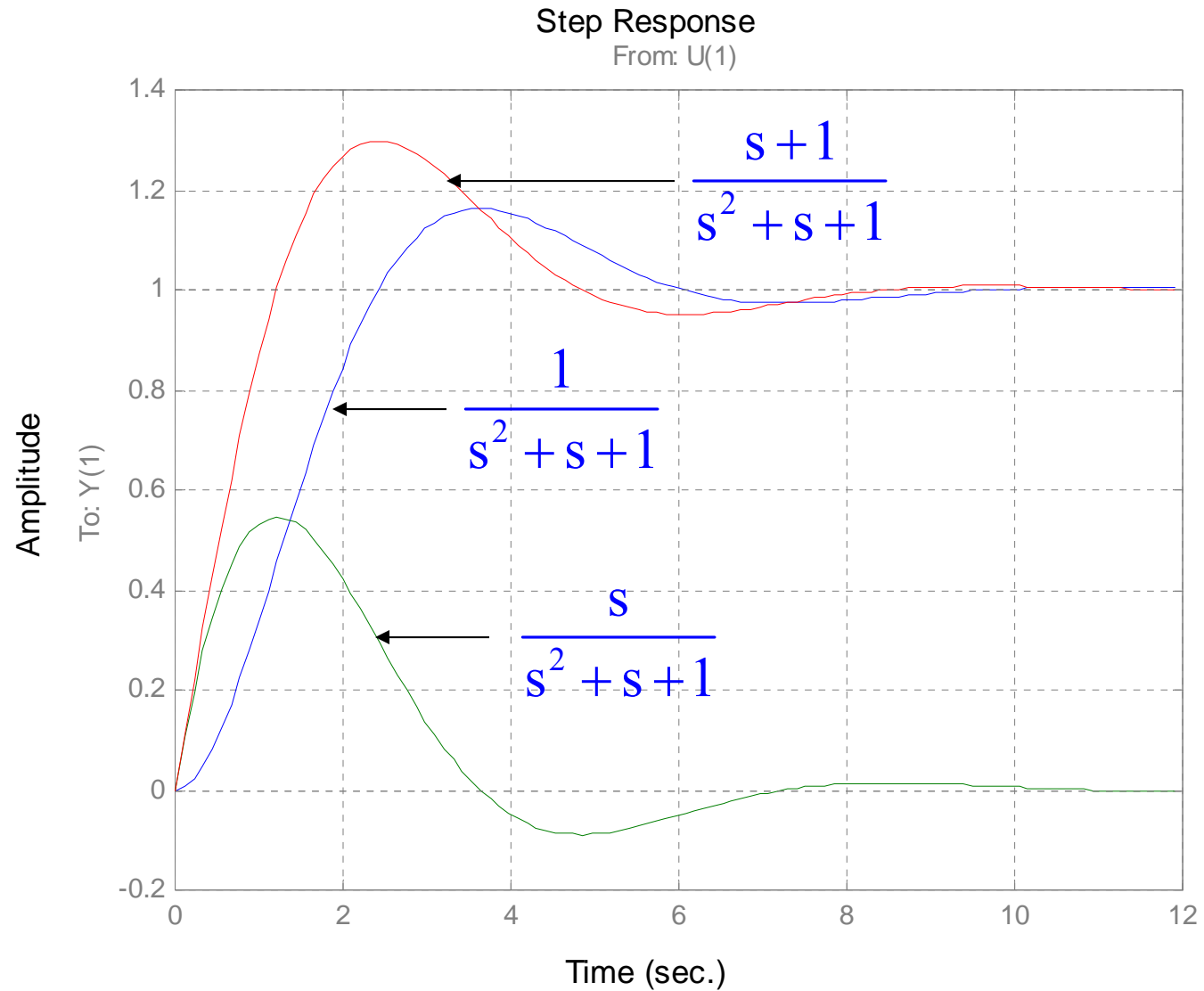
- This does not hold for a nonminimum-phase system.
- Nonminimum-phase systems may arise in two different ways:
 - When a system includes a nonminimum-phase element or elements
 - When there is an unstable minor loop
- For a minimum-phase system, the phase angle at $\omega = \infty$ becomes $-90^\circ(q - p)$, where p and q are the degrees of the numerator and denominator polynomials of the transfer function, respectively.
- For a nonminimum-phase system, the phase angle at $\omega = \infty$ differs from $-90^\circ(q - p)$.
- In either system, the slope of the log magnitude curve at $\omega = \infty$ is equal to $-20(q - p)$ dB/decade.

- It is therefore possible to detect whether a system is minimum phase by examining both the slope of the high-frequency asymptote of the log-magnitude curve and the phase angle at $\omega = \infty$. If the slope of the log-magnitude curve as $\omega \rightarrow \infty$ is $-20(q - p)$ dB/decade and the phase angle at $\omega = \infty$ is equal to $-90^\circ(q - p)$, then the system is minimum phase.
- Nonminimum-phase systems are slow in response because of their faulty behavior at the start of the response.
- In most practical control systems, excessive phase lag should be carefully avoided. A common example of a nonminimum-phase element that may be present in a control system is transport lag: $e^{-\tau_{dt}s} = 1 \angle -\omega\tau_{dt}$

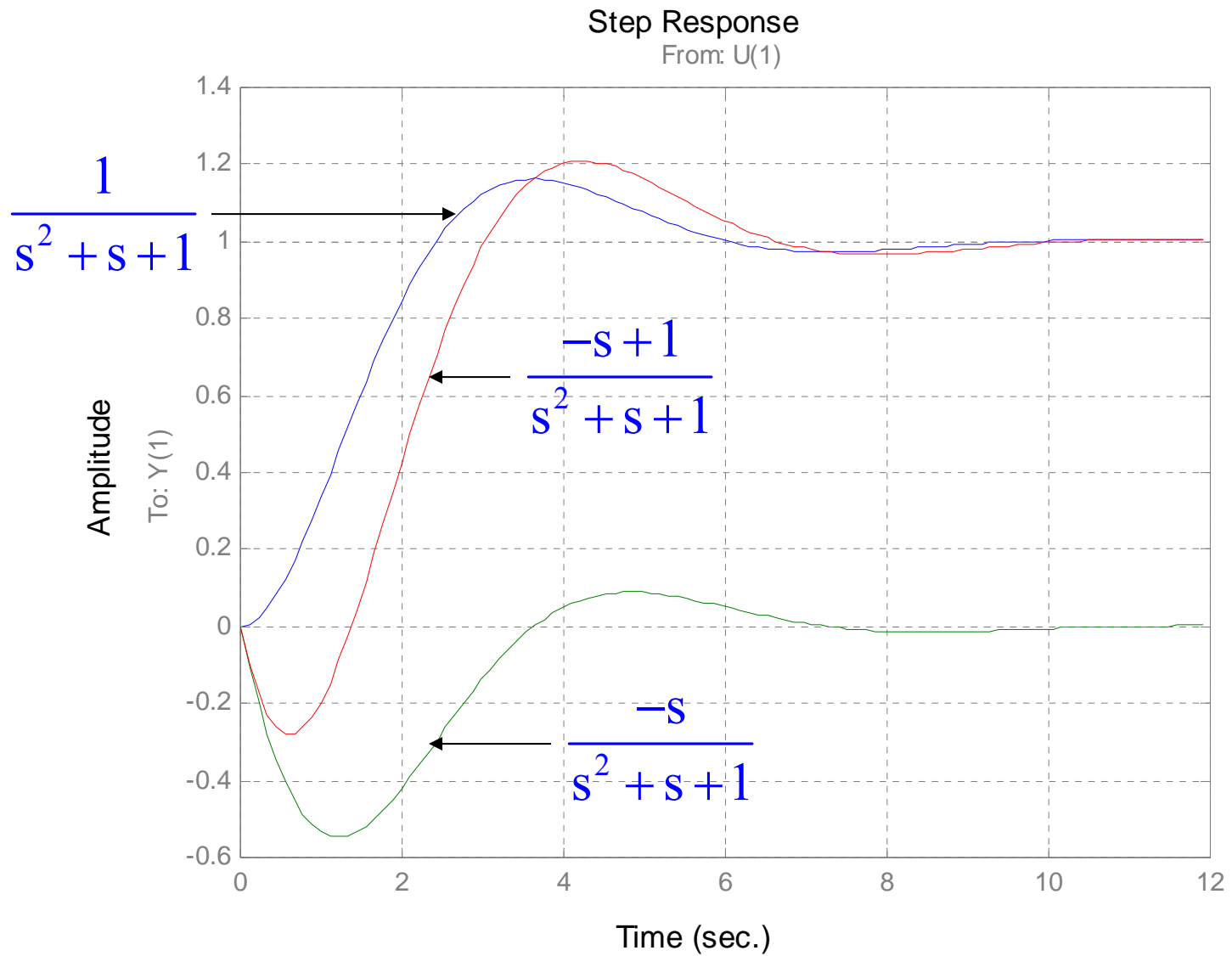
Unit Step Responses



Unit Step Responses



Unit Step Responses

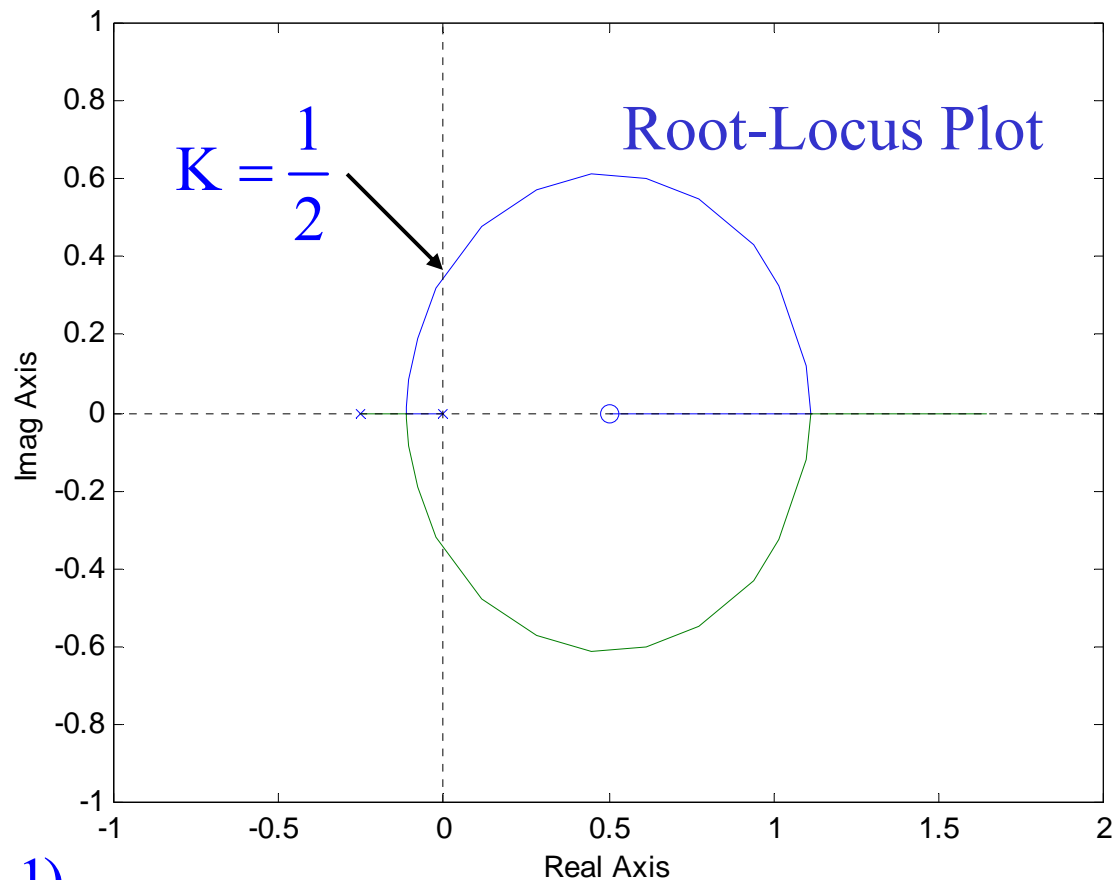


- Nonminimum-Phase Systems: Root-Locus View

- If all the poles and zeros of a system lie in the LHP, then the system is called *minimum phase*.
- If at least one pole or zero lies in the RHP, then the system is called *nonminimum phase*.
- The term nonminimum phase comes from the phase-shift characteristics of such a system when subjected to sinusoidal inputs.
- Consider the open-loop transfer function:

$$G(s)H(s) = \frac{K(1-2s)}{s(4s+1)}$$

$$G(s)H(s) = \frac{K(1-2s)}{s(4s+1)}$$



Angle Condition:

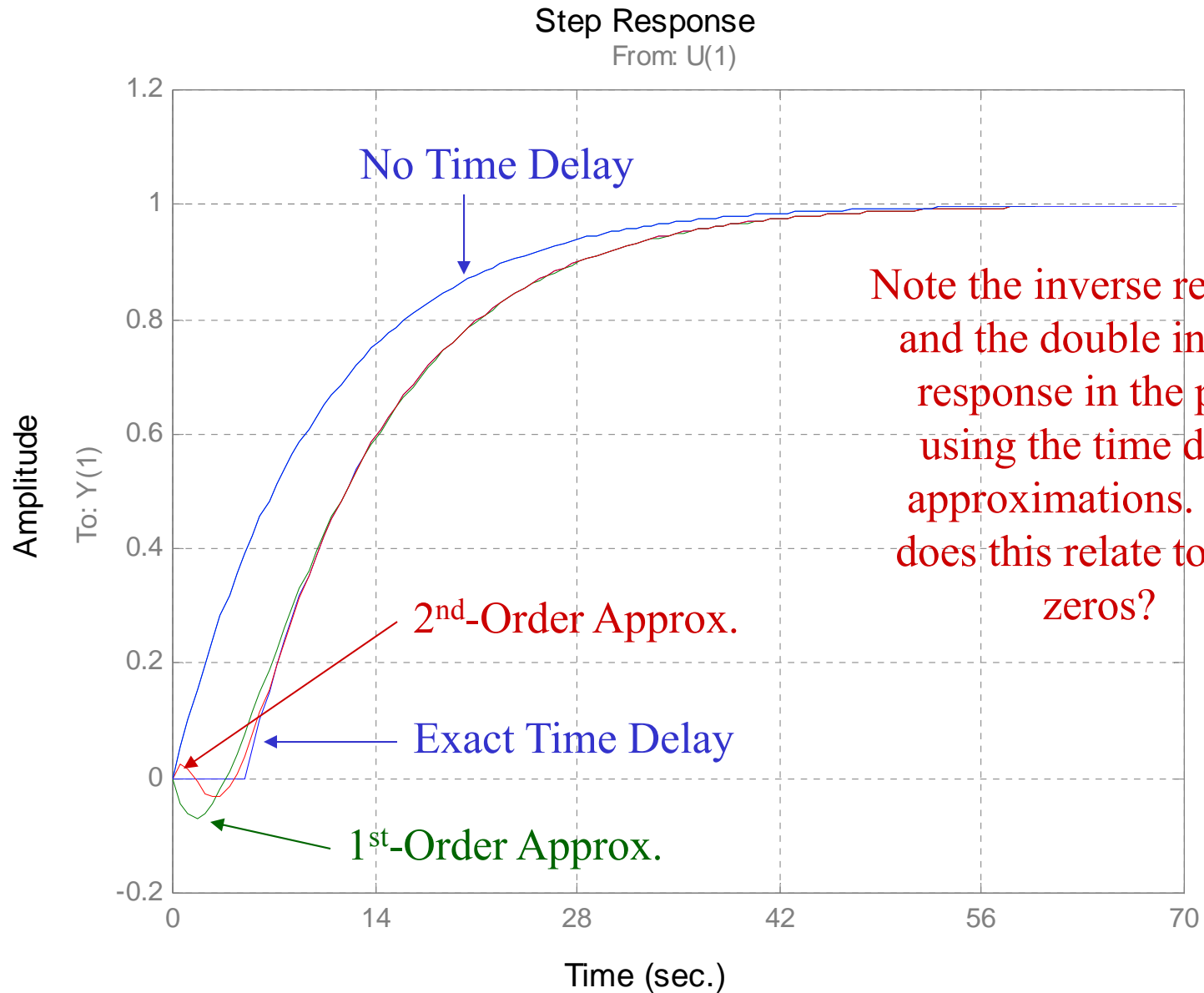
$$\begin{aligned} \angle G(s)H(s) &= \angle \frac{-K(2s-1)}{s(4s+1)} \\ &= \angle \frac{K(2s-1)}{s(4s+1)} + 180^\circ = \pm 180^\circ(2k+1) \quad \text{or} \quad \angle \frac{K(2s-1)}{s(4s+1)} = 0^\circ \end{aligned}$$

- Dynamic Response of a First-Order System with a Time Delay

- The transfer function of a time delay combined with a first-order process is:

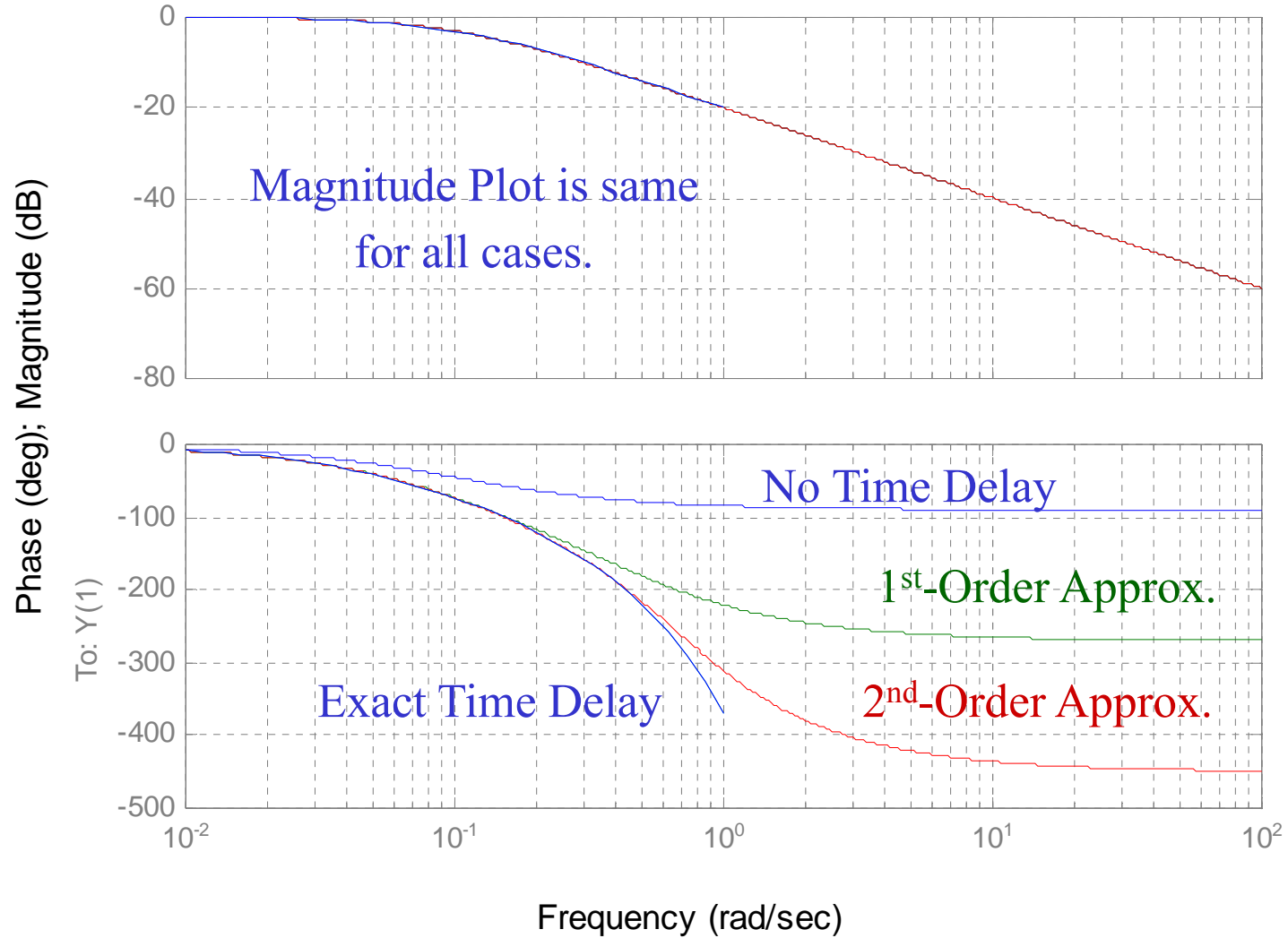
$$\frac{Ke^{-\tau_{DT}s}}{\tau s + 1}$$

- Consider the case with: $K = 1$, $\tau = 10$, $\tau_{DT} = 5$, and a unit step input at $t = 0$.
- Simulate the step response with:
 - An exact representation of a time delay
 - A first-order Pade approximation of a time delay
 - A second-order Pade approximation of a time delay
- Simulate the frequency response for the same cases.



Bode Diagrams

From: U(1)

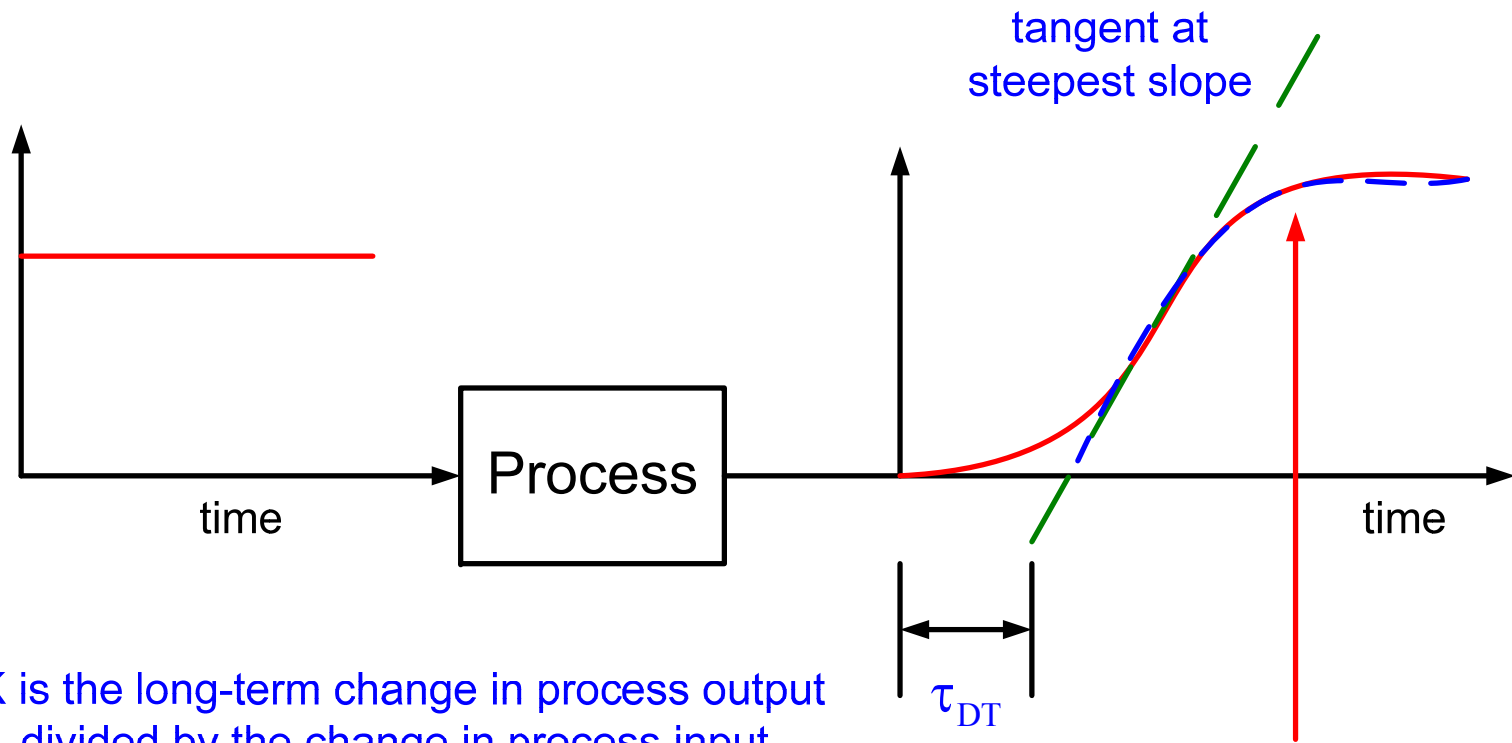


- Empirical Model
 - The most common plant test used to develop an empirical model is to make a step change in the manipulated input and observe the measured process output response.
 - Then a model is developed to provide the best match between the model output and the observed plant output.
 - Important Issues:
 - Selection of the proper input and output variables.
 - In step-response testing, we first bring the process to a consistent and desirable steady-state operating point, then make a step change in the input variable.
 - What should the magnitude of the step change be?

1. The magnitude of the step input must be large enough so that the output signal-to-noise ratio is high enough to obtain a good model.
 2. If the magnitude of the step input is too large, nonlinear effects may dominate.
 - Clearly there is a trade off.
- By far the most commonly used model for control-system design purposes, is the 1st-order plus time delay model.

$$\frac{Ke^{-\tau_{DT}s}}{\tau s + 1}$$

- The three process parameters can be estimated by performing a single step test on the process input.



K is the long-term change in process output divided by the change in process input.

Estimate time constant from a semi-log plot of first-order response.

- If the process is already in existence, experimental step tests allow measurement of τ_{DT} and τ .
- At the process design stage, theoretical analysis allows estimation of these numbers if the process is characterized by a cascade of known 1st-order lags.
- Approximate the dead time with a 1st-order Pade approximation:

$$\frac{2 - \tau_{DT}S}{2 + \tau_{DT}S}$$
- Consider the open-loop transfer function:

$$\frac{Ke^{-\tau_{DT}S}}{\tau S + 1} \approx \frac{K \left(\frac{2 - \tau_{DT}S}{2 + \tau_{DT}S} \right)}{\tau S + 1} = G$$

- The closed-loop system transfer function is:

$$\frac{C}{V} = \frac{G}{1+G}$$

- The characteristic equation of the closed-loop system is:

$$1 + G(s) = 0$$

$$1 + \frac{K \left(\frac{2 - \tau_{DT} s}{2 + \tau_{DT} s} \right)}{\tau s + 1} = 0$$

$$\tau_{DT} \tau s^2 + (2\tau + \tau_{DT} - K\tau_{DT})s + 2(K + 1) = 0$$

- For what value of K will this system go unstable?

- The Routh Stability Criterion predicts that for stability:

$$-1 < K < 2 \left(\frac{\tau}{\tau_{DT}} \right) + 1$$

- The gain value for marginal stability can be found precisely from the Nyquist criterion since we know the frequency response of a dead time exactly. For marginal stability, we require that $(B/E)(i\omega)$ go precisely through the point $-1 = 1 \angle 180^\circ$. The phase angle part of the requirement can be stated as:

$$-\pi = -\omega_0 \tau_{DT} - \tan^{-1} \omega_0 \tau$$

- This fixes (for a given τ τ_{DT}) the frequency ω_0 at which $(B/E)(i\omega)$ passes through the point $-1 = 1 \angle 180^\circ$.

- This equation has no analytical solution. Once ω_0 is found numerically, the gain K for marginal stability is obtained by requiring that:

$$\left| \frac{B}{E}(i\omega) \right| = \frac{K}{\sqrt{(\omega_0\tau)^2 + 1}} = 1.0$$

- A table shows results for a range of the most common values encountered for τ_{DT} / τ in modeling complex systems.

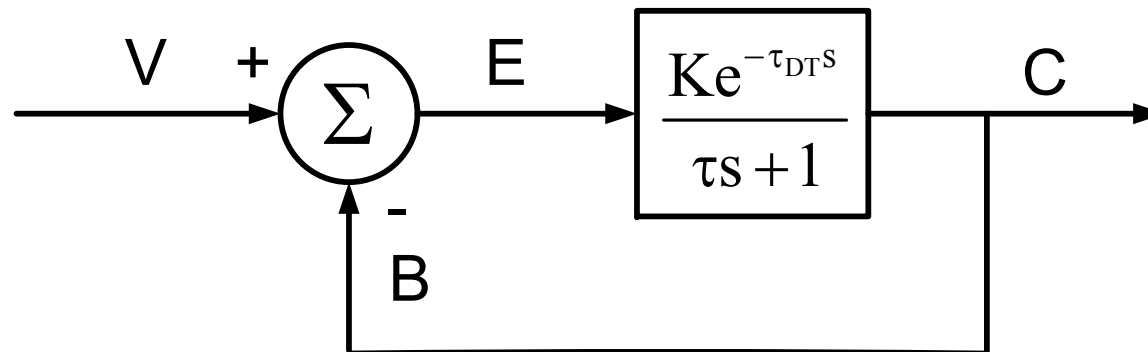
τ_{DT} / τ	$\omega_0 \tau$	K
0.1	16.4	16.4
0.2	8.44	8.50
0.3	5.80	5.89
0.4	4.48	4.59
0.5	3.67	3.81
0.6	3.13	3.29
0.7	2.74	2.92
0.8	2.45	2.64
0.9	2.22	2.43
1.0	2.03	2.26

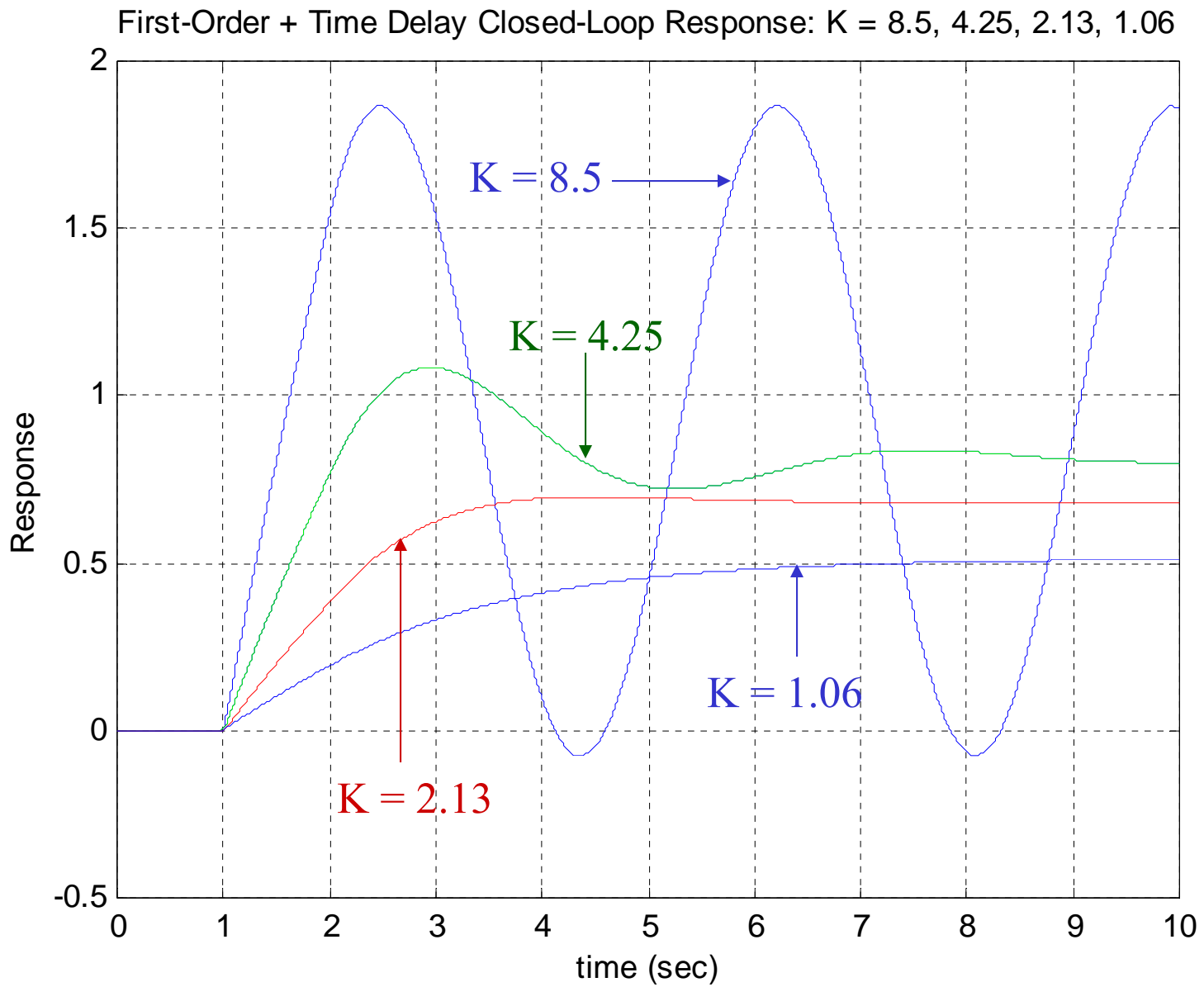
- The steady-state error is typical of proportional control. Design values of K must be less than those for marginal stability.
- A design criterion sometimes used in industrial process control is quarter-amplitude damping, wherein each cycle of transient oscillation is reduced to $\frac{1}{4}$ the amplitude of the previous cycle.
- A useful approximation for this behavior is a gain margin of 2.0 for the frequency response.
- If we apply this to the table of results for, say, $\tau_{DT} / \tau = 0.2$, we get a design gain value of 4.25, giving large steady-state errors.
- For this reason, processes of this type often use integral or proportional + integral control, which reduces steady-state errors without requiring large K values.

- Exercise:

- For the closed-loop system below, evaluate the step response using:

- $\tau_{DT} = 1 \text{ sec}$
- $\tau = 5 \text{ sec}$
- $K = 8.5, 4.25, 2.13, 1.06$

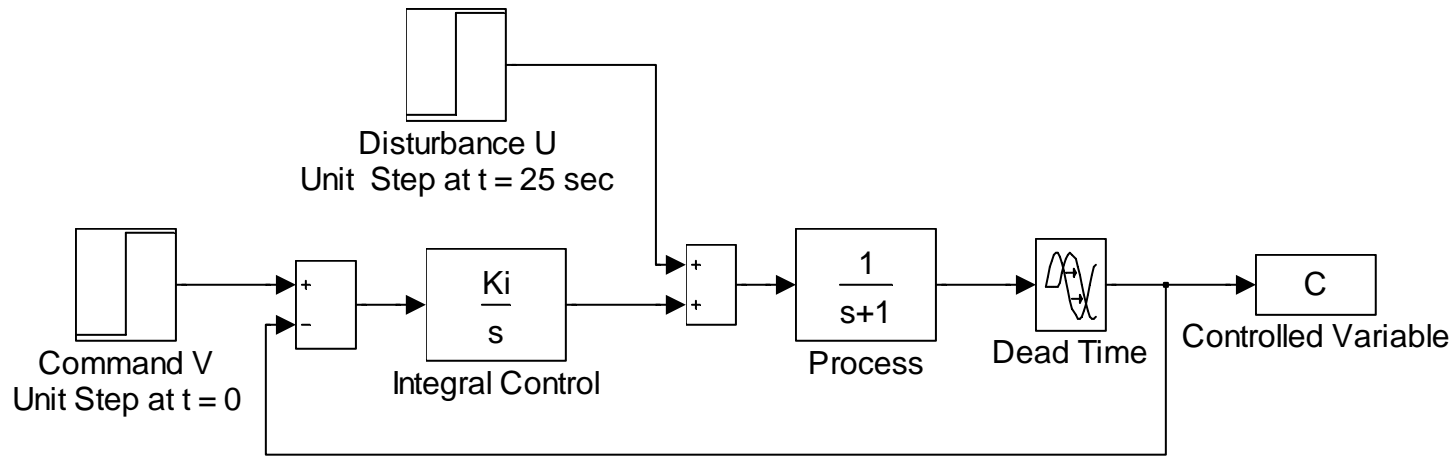




- Consider Integral Control of a First-Order Process plus a Dead Time
 - Proportional control was found to be difficult since loop gain was restricted by stability problems to low values, causing large steady-state error.
 - Integral control gives zero steady-state error (for both step commands and/or disturbances) for any loop gain and is thus an improvement.
 - The values of K for marginal stability are given in the following table.
 - Compared with proportional control, both loop gain and speed of response (ω_0 for a given τ) are lower. However, we do not depend on it to reduce steady-state error.

τ_{DT} / τ	$\omega_0 \tau$	K
0.1	3.11	10.2
0.2	2.16	5.16
0.3	1.74	3.49
0.4	1.48	2.65
0.5	1.31	2.15
0.6	1.18	1.81
0.7	1.07	1.57
0.8	0.99	1.39
0.9	0.92	1.25
1.0	0.86	1.14

- Check Time-Domain Response



- Run simulations on the system for $K_I = 1.14$ (marginal stability) and for $K_I = 0.57$ (gain margin of 2.0).
- Check response of C to both step inputs in V and U.
- Note the well-damped response with zero steady-state error for both step commands and disturbances for $K_I = 0.57$.

Integral Control: First-Order + Time Delay Closed-Loop Response: $K_I = 1.14, 0.57$

