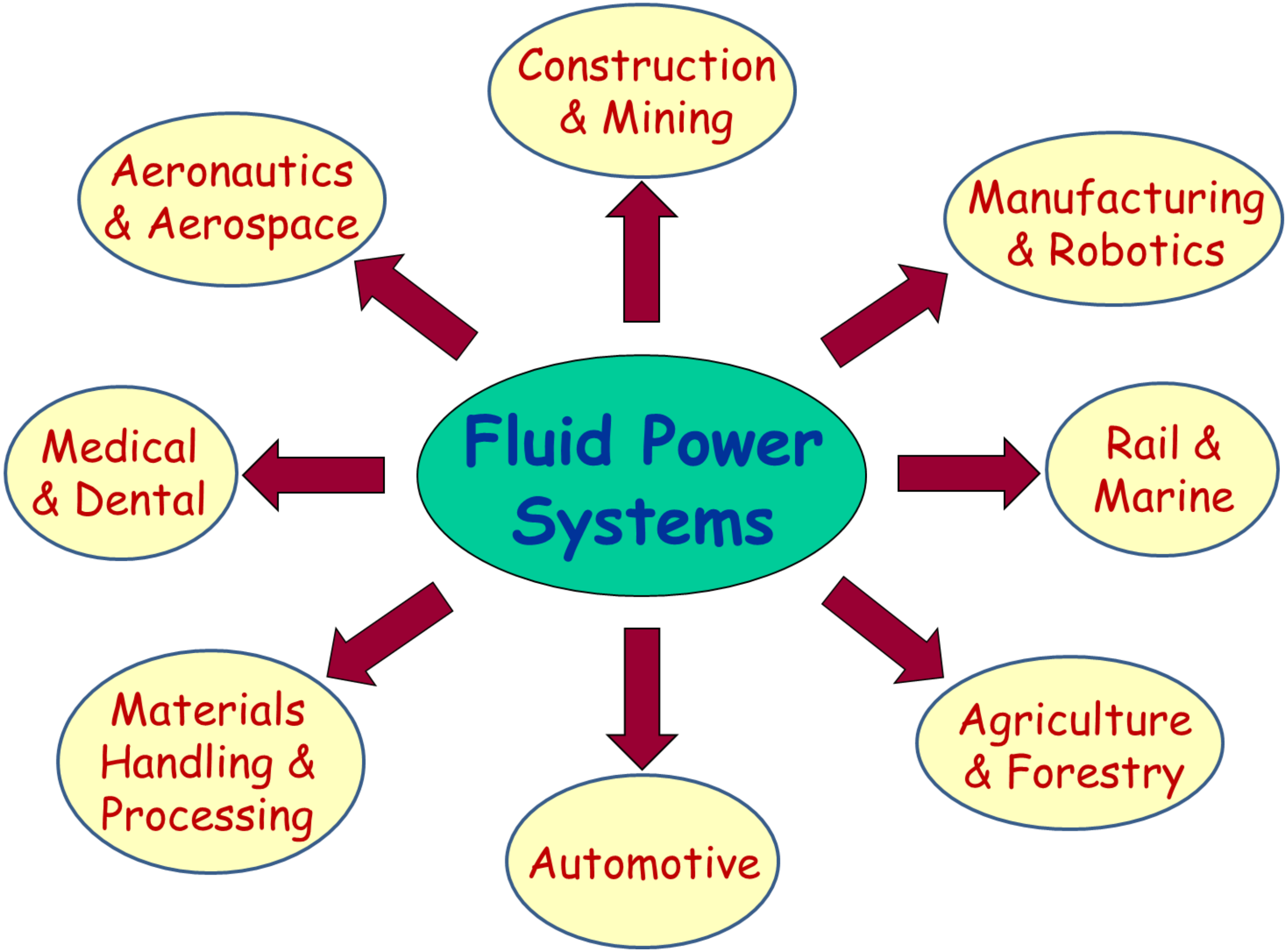
Fluid Power Systems & Control



Hydraulic Transmission Lines

- One of the most neglected aspects of hydraulic circuit design is performance under dynamic conditions.
- Under steady flow conditions, understanding resistive characteristics of components and piping is required.
- However, the system has energy storage characteristics in the form of fluid and mechanical mass, together with structural and fluid elasticity. These characteristics are very sensitive to the frequency content of the transmitted power.
- There are strong analogies that can be drawn between fluid and electrical power systems under transient conditions, allowing the fluid power engineer to take advantage of the well-established theories and techniques of the electrical engineer.

Flow Networks

- Any interconnection of elements and sources is called a network.
- The elements are the valves and cylinders, the sources are the pumps and compressors, and the network connections are the pipework.
- Flow and pressure distributions within such a network must satisfy three basic constraints:
  - The flow-pressure relationship (constitutive equation) for the elements. This is the equation relating the input to the output across a device.

- The equivalent of Kirchhoff's current law: If q_i represents the flow through the i^{th} pipe of the network, then at any junction of pipes: $\sum q_i = 0$
- The equivalent of Kirchhoff's voltage law: If δP_i is the pressure drop across the i^{th} pipe of the network then around a closed circuit: $\sum \delta P_i = 0$
- Kirchhoff's Flow Law states that the algebraic sum of the flows entering a connection point is zero. A connection point is a point in the flow network at which one or more elements or sources terminate.
- Kirchhoff's Pressure Law states that the algebraic sum of the pressure drops around a closed path is zero.

Classification of System Models

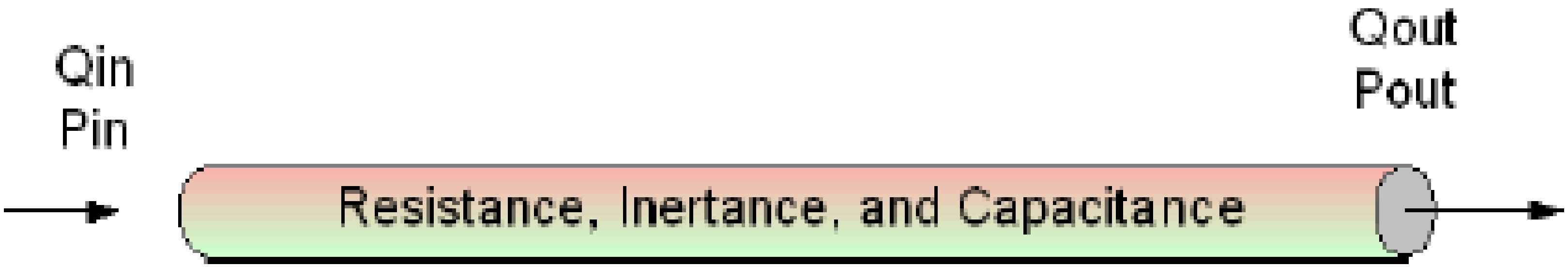
Nonlinear	Principle of Superposition Does Not Apply	Nonlinear Differential Equations
Linear	Principle of Superposition Applies	Linear Differential Equations
Distributed	Dependent Variables are Functions of Space & Time	Partial Differential Equations
Lumped	Dependent Variables are Functions Only of Time	Ordinary Differential Equations
Time-Varying	Model Parameters Vary in Time	Equations with Time-Varying Parameters
Stationary	Model Parameters are Constant in Time	Equations with Constant Parameters
Continuous	Dependent Variables defined over Continuous Range of Independent Variables	Differential Equations
Discrete	Dependent Variables defined only for Distinct Values of Independent Variables	Time-Difference Equations

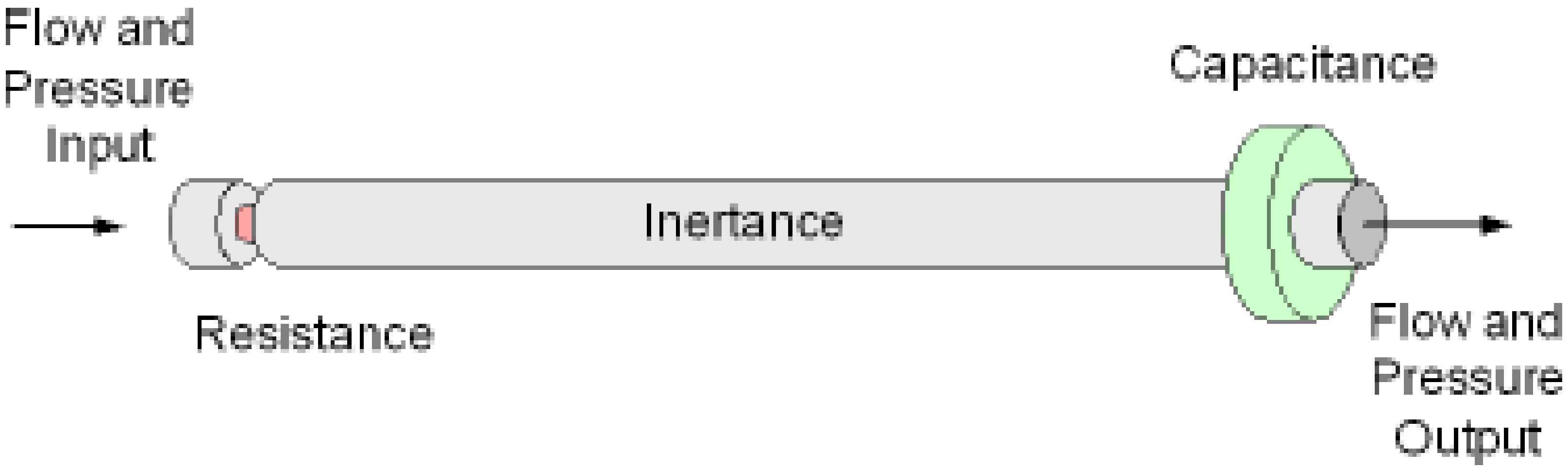
Analogies Ideal Linear System Elements

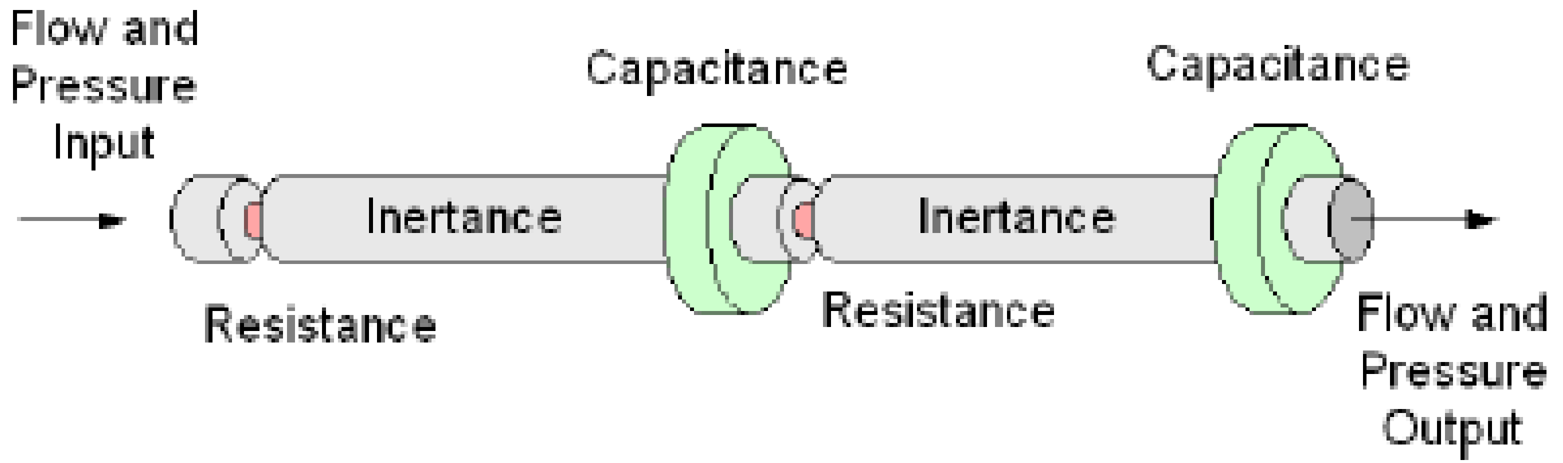
System Type	Electrical	Fluid
A-type element	Capacitor	Fluid capacitor
Elemental equation	$i = C\frac{de}{dt}$	$Q_f = C_f\frac{dP}{dt}$
Energy stored	Electric field	Potential
Energy equation	$\mathscr{E}_E = \frac{C}{2}e^2$	$\mathscr{E}_P = \frac{C_f}{2}P^2$
T-type element	Inductor	Inertor
Elemental equation	$e = L\frac{di}{dt}$	$P = I\frac{dQ_f}{dt}$
Energy stored	Magnetic field	Kinetic
Energy equation	$\mathscr{E}_M = \frac{L}{2}i^2$	$\mathscr{E}_K = \frac{I}{2}Q_f^2$
D-type element	Resistor	Fluid resistor
Elemental equation	$i = \left(\frac{1}{R}\right)e$	$Q_f = \left(\frac{1}{R_f}\right)P$
	$e = Ri$	$P = R_fQ_f$
Energy dissipation rate	$\frac{d\mathscr{E}_D}{dt} = ie$	$\frac{d\mathscr{E}_D}{dt} = Q_fP$

Lumped Parameter Modeling

- The actual governing equations of a fluid transmission line are nonlinear partial differential equations that model the distributed nature of the three parameters of resistance, inertance, and capacitance.




Flow and
Pressure
Input
Capacitance
Inertance
Resistance
Flow and
Pressure
Output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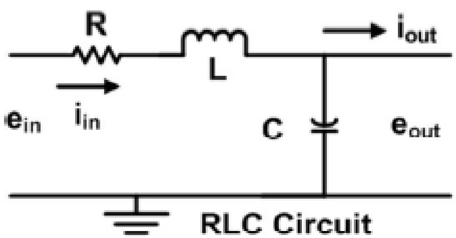
Hydraulic Transmission Lines

Neglecting Transmission Lines Can Lead to Poor Dynamic Performance & Instability

In applications where large forces / torques are required, with a fast response time and high accuracy, hydraulic control systems are essential. They have a more competitive power-weight ratio than electrically-actuated systems, which are limited by magnetic saturation, and they excel in environmentally-difficult applications. In addition, the hydraulic medium is mechanically stiffer than the electromagnetic medium. Self-lubrication and inherent heat transfer are also advantages. Fluid power applications are numerous, e.g., vehicle steering, braking, and suspension systems; industrial mechanical manipulators and robots; and actuators for aircraft and marine vessels, to name a few. They are all multidisciplinary systems and require a systems approach to design and implement with optimal energy consumption. Hydraulic valves, pumps, motors, cylinders, and accumulators are all routinely modeled and analyzed. Often overlooked in a hydraulic control system are the hydraulic transmission lines and failure to model their dynamic effect could lead to poor performance and possible instability.

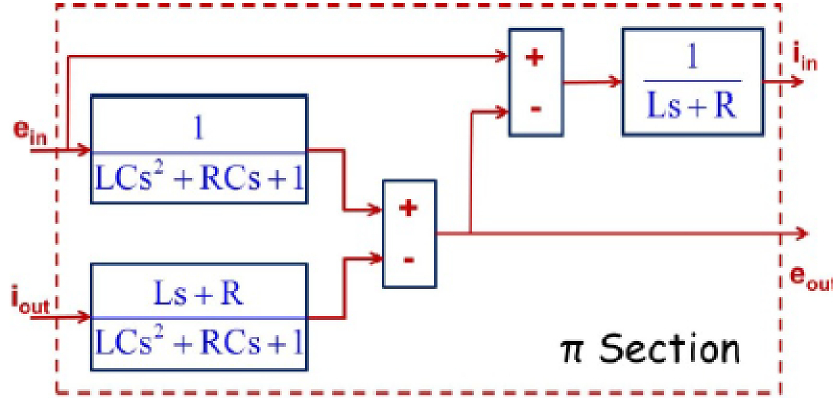
The hydraulic transmission line is a distributed system. The motion of the fluid in the transient condition takes place under the action of fluid inertia, friction, and compressibility, as well as the driving pressure forces. Fluid velocity, pressure, and temperature vary from point to point along the pipe / hose length and radius, which may itself be compliant. A lumped-parameter model which describes the dynamic behavior of the transmission line with acceptable accuracy is needed.

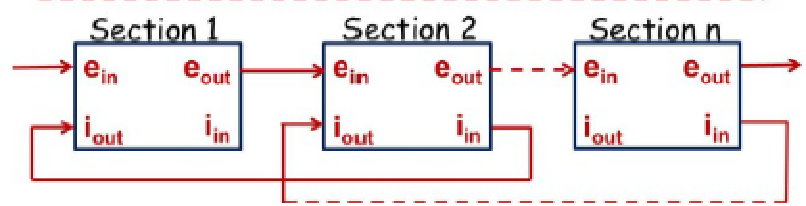
It is insightful to use the electric-hydraulic analogy to develop and understand the lumped-parameter model. The effort variables are electric voltage and fluid pressure, while the flow variables are electric current and fluid flow rate. Energy is stored via the effort variables in the electrical capacitor and fluid capacitance. Energy is stored via the flow variables in the electrical inductor and fluid inertia. Energy is dissipated in the electrical and fluid resistance elements. Fluid resistance and fluid inertia are generally well understood. Fluid capacitance is due to the bulk modulus (measure of fluid compressibility) of the fluid, which is reduced by entrained air, and the compliance of the flexible hose. The analog RLC electrical circuit is shown and the input-output matrix is derived by applying Kirchhoff's Laws, applicable to both electrical and hydraulic circuits.



$$\begin{bmatrix} e_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} LCs^2 + RCs + 1 & Ls + R \\ Cs & 1 \end{bmatrix} \begin{bmatrix} e_{out} \\ i_{out} \end{bmatrix}$$

A block diagram for one lumped element, called a π section because of its circuit shape, is shown. This applies to both the electrical and hydraulic circuits. This is easily implemented in MatLab / Simulink and gives the same results as obtained using the predefined electrical and hydraulic transmission line blocks in MatLab





SimScape. This approach, however, gives more insight and understanding. The greater the number of sections used, the closer the dynamic behavior prediction is to that obtained from the distributed-parameter model, and more importantly, to the actual system behavior.

How are the hydraulic resistance, inertia, and capacitance determined for each section of the transmission line? It is common to assume laminar, unidirectional flow using the average pressure and velocity at any cross section and at any instant. The pressure change-flow rate relationship is then linear and algebraic, with the resistance R as the proportionality constant. The lump of fluid in a section can be treated as a rigid body to which we apply Newton's 2^{nd} Law to get the relationship between pressure change and the rate of change of flow rate, with the fluid inertance I, as it is called, as the proportionality constant. The capacitance C of the fluid lump shows the relationship between volume change and pressure change, and is due to the compressibility of the fluid, represented by its bulk modulus, and modified to include the effects of entrained air and flexible hoses.

Hydraulic transmission lines, like electrical transmission lines, can be modeled in a lumped-parameter way and combined with models of other components to predict dynamic response and stability and for control design.