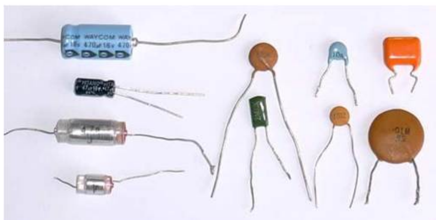
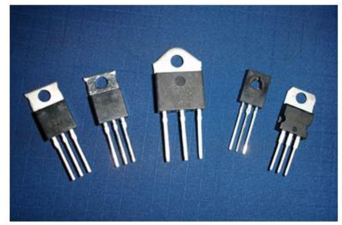
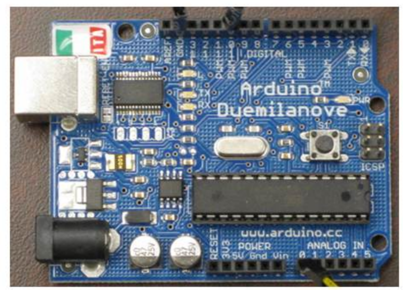
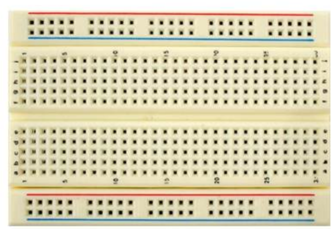
Op-Amp Example Problems with Solutions



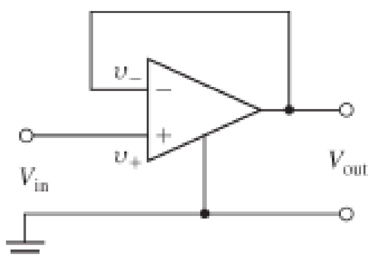




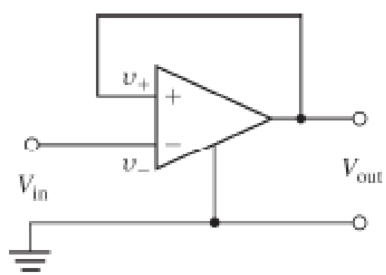
For the following electrical system problems, the non-ideal op-amp model to be used is given below, where A is a very large number ($\approx 10^7$) and τ is the time constant.

$$V_{out} = \frac{A}{\tau s + 1}\left[V^+ - V^-\right] \qquad i^+ = i^- = 0$$

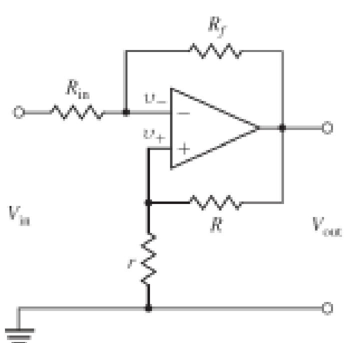
1. Show that the op-amp connection shown results in $V_{out} = V_{in}$ if the op-amp is ideal. Give the transfer function if the op-amp has the non-ideal model.



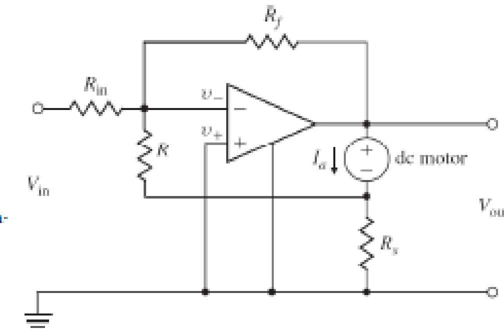
2. Show that, with the non-ideal op-amp model, the op-amp connection shown is unstable.



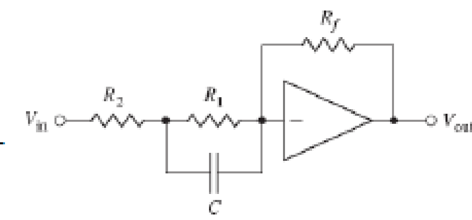
3. An op-amp connection with feedback to both the negative and positive terminals is shown. If the op-amp has the non-ideal transfer function, give the maximum value possible for the positive feedback ratio, P = r/(r+R), in terms of the negative feedback ratio, $N = R_{in}/(R_{in}+R_f)$, for the circuit to remain stable.



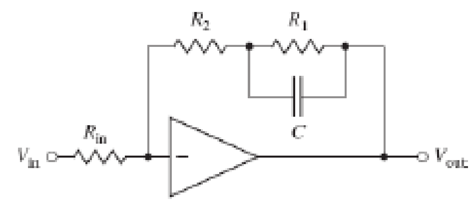
4. A common connection for a motor power amplifier is shown. The idea is to have the motor current follow the input voltage, and the connection is called a current amplifier. Model the motor as an inductor L_M and resistor R_M in series. Assume that the sense resistor R_S is very small compared with the feedback resistor R. For the ideal op-amp model, find the transfer function from V_{in} to I_a. Also show the transfer function when $R_f = \infty$.



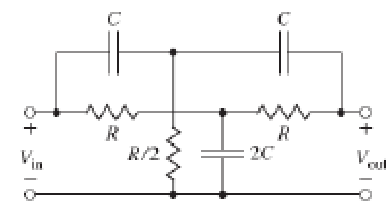
5. Write the dynamic equations and find the transfer functions for the circuits shown: (a) lead circuit, (b) lag circuit, and (c) notch circuit. Use the ideal op-amp model.



(a)

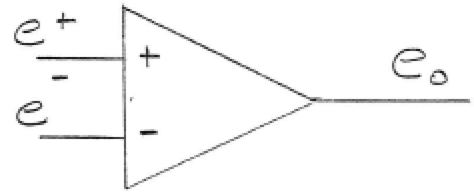


(b)



(c)

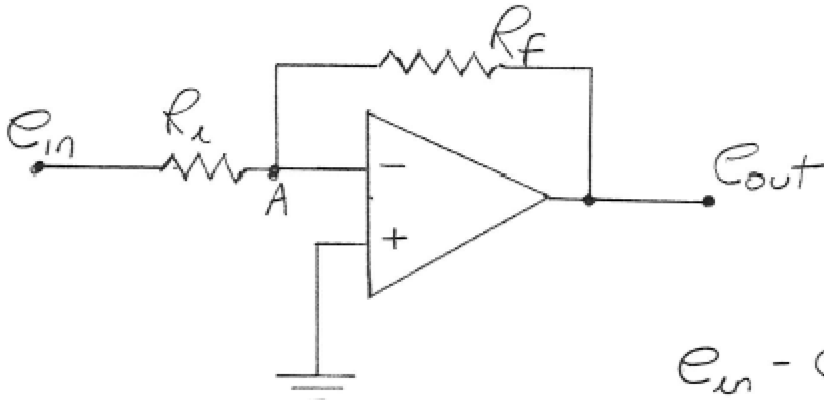
$\boxed{1.}$



$$e_o = \frac{A}{\tau s + 1}\left(e^+ - e^-\right)$$

$$i^+ = i^- = 0$$

Realistic Op-Amp Model



Inverting Op-Amp

Ideal Assumptions

$$e^+ = e^- \qquad i^+ = i^- = 0$$

$$\frac{e_{in} - 0}{R_i} + \frac{e_{out} - 0}{R_f} = 0 \qquad \text{KCL Node A}$$

$$\frac{e_o}{e_i} = -\frac{R_f}{R_i}$$

Non-Ideal

$$\frac{e_i - e^-}{R_i} + \frac{e_o - e^-}{R_f} = 0$$

$$e^- = \frac{e_i R_f + e_o R_i}{R_i + R_f} \Longrightarrow e_o = \frac{A}{\tau s + 1}\left(e^+ - e^-\right)$$

$$e_o = \frac{A}{\tau s + 1}\left(0 - \frac{e_i R_f + e_o R_i}{R_i + R_f}\right)$$

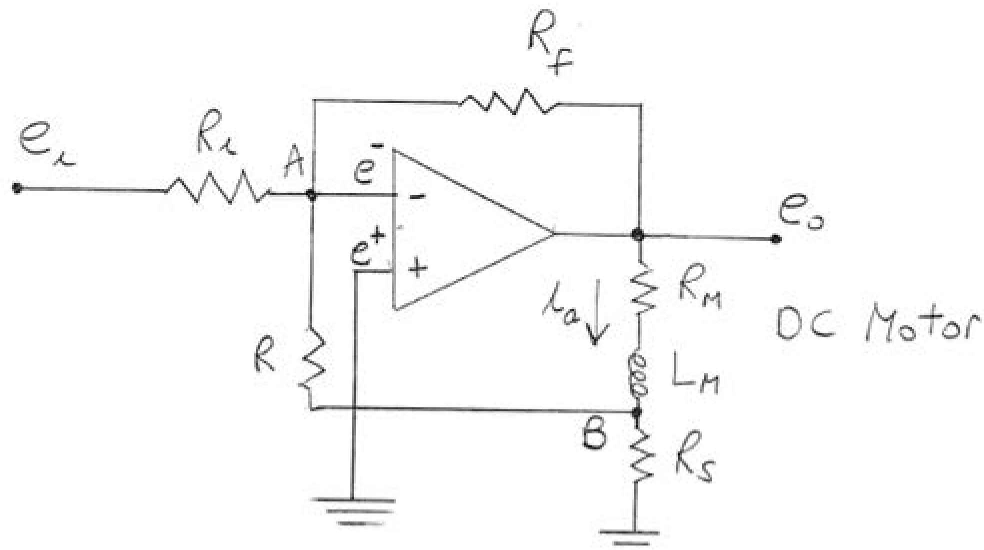
Solve for e_o :

$$\frac{e_o}{e_i} = \frac{-A\left(\frac{R_f}{R_i + R_f}\right)}{\tau s + 1 + A\left(\frac{R_i}{R_i + R_f}\right)}$$

For : $A \to \infty$, $\tau \to 0$

$$\frac{e_o}{e_i} = -\frac{R_f}{R_i}$$

$\boxed{2.}$ Current Amplifier (Motor current follows input voltage)



Assume:

$R_s << R$

Ideal Op-Amp

$e^+ = e^-$

$i^+ = i^- = 0$

KCL Node A:

$$e_A = e^- = e^+ = 0 \qquad \frac{e_i - 0}{R_i} + \frac{e_o - 0}{R_f} + \frac{e_B - 0}{R} = 0$$

$$\frac{e_i}{R_i} + \frac{e_o}{R_f} + \frac{e_B}{R} = 0$$

KCL Node B:

$$i_a + \frac{0 - e_B}{R} + \frac{0 - e_B}{R_s} = 0$$

$$e_B = \frac{R R_s}{R + R_s} i_a = \frac{R_s}{1 + R_s/R} i_a \approx R_s i_a$$

for $R_s << R$

DC Motor Dynamics

$$\left.\begin{aligned} J_m \ddot{\theta}_m + b \dot{\theta}_m &= K_t i_a \\ J_m \dot{\omega}_m + b \omega_m &= K_t i_a \\ (J_m s + b)\omega_m &= K_t i_a \end{aligned}\right\} \text{Mechanical} \qquad \omega_m = \frac{K_t i_a}{J_m s + b}$$

$(\omega_m = \dot{\theta}_m)$

$$\left.\begin{aligned} e_a - i_a R_a - \dot{i}_a L_a - K_e \omega_m &= 0 \\ e_a &= \dot{i}_a L_a + i_a R_a + K_e \omega_m \\ (L_a s + R_a) i_a + K_e \omega_m &= e_a \end{aligned}\right\} \text{Electrical}$$

$$e_o - e_B = e_a \text{ (motor armature voltage)}$$

$$e_o - R_s i_a = e_a$$

$$e_o = i_a R_s + (L_a s + R_a) i_a + K_e \omega_m$$

$$e_o = i_a R_s + (L_a s + R_a) i_a + K_e \left(\frac{K_t i_a}{J_m s + b} \right)$$

Let's go back to KCL for Node A:

$$\frac{e_i}{R_i} + \frac{e_o}{R_f} + \frac{e_B}{R} = 0$$

$$\frac{e_i}{R_i} + \frac{i_a}{R_f} \left[L_a s + R_a + R_s + \frac{K_e K_t}{J_m s + b} \right] + \frac{R_s i_a}{R} = 0$$

In steady state $s \to 0$

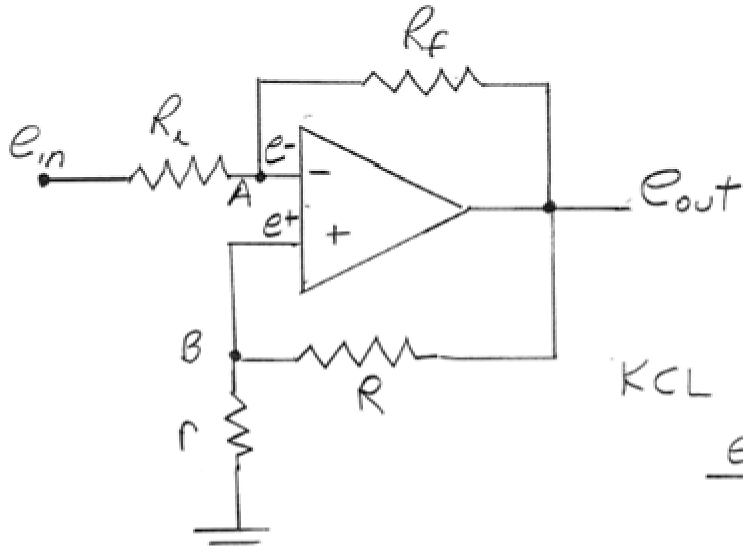
$$\frac{e_i}{R_i} + \frac{i_a}{R_f} \left[R_a + R_s + \frac{K_e K_t}{b} \right] + \frac{R_s i_a}{R} = 0$$

$$\frac{e_i}{R_i} + \left[\frac{R_a}{R_f} + \frac{R_s}{R_f} + \frac{K_e K_t}{b R_f} + \frac{R_s}{R}\right] i_a = 0$$

If $R_f \to \infty$ Then $$\frac{e_i}{R_i} + \frac{R_s i_a}{R} = 0$$

$$\frac{i_a}{e_i} = -\frac{R}{R_i R_s}$$

$\boxed{3.}$



Non-Ideal TF

$$e_o = \frac{A}{\tau s + 1}\left(e^+ - e^-\right)$$

KCL Node A

$$\frac{e_i - e^-}{R_i} + \frac{e_o - e^-}{R_f} = 0$$

KCL Node B

$$\frac{e_o - e^+}{R} + \frac{0 - e^+}{r} = 0$$

Define: $$\rho = \frac{r}{r+R}$$

$$N = \frac{R_i}{R_i + R_f}$$

$$\frac{e_i - e^-}{R_i} + \frac{e_o - e^-}{R_f} = 0 \Rightarrow \frac{e_i}{R_i} + \frac{e_o}{R_f} = e^-\left(\frac{1}{R_i} + \frac{1}{R_f}\right)$$

$$e^- = \frac{R_f}{R_i + R_f} e_i + \frac{R_i}{R_i + R_f} e_o$$

$$e^- = (1-\nu)e_i + \nu e_o$$

$$\frac{e_o - e^+}{R} + \frac{0 - e^+}{r} = 0 \Rightarrow \frac{e_o}{R} = e^+\left(\frac{1}{R} + \frac{1}{r}\right)$$

$$e^+ = \frac{r}{R+r} e_o = \rho e_o$$

$$e_o = \frac{A}{\tau s+1}\left(e^+ - e^-\right) \Rightarrow e_o = \frac{A}{\tau s+1}\left(\rho e_o - (1-\nu)e_i - \nu e_o\right)$$

$$e_o = \frac{A}{\tau s+1}\left((\rho-\nu)e_o - (1-\nu)e_i\right)$$

$$e_o\left[1 - \frac{A}{\tau s+1}(\rho-\nu)\right] = \frac{-A}{\tau s+1}(1-\nu)e_i$$

$$\frac{e_o}{e_i} = \frac{A(\nu-1)}{\tau s+1 - A(\rho-\nu)}$$

<u>Stability</u>

$$1 - A(\rho-\nu) > 0$$

$$1 > A(\rho-\nu)$$

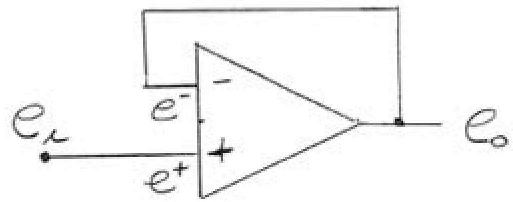
$$A\nu - A\rho > -1$$

$$\nu - \rho > -\frac{1}{A}$$

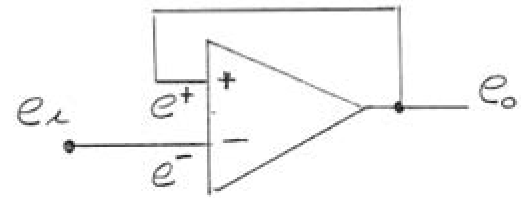
$$\rho < \nu + \frac{1}{A}$$

4.

Stable



Unstable



$$e_o = \frac{A}{\tau s+1}\left(e^+ - e^-\right)$$

$$e_o = \frac{A}{\tau s+1}\left(e_i - e_o\right)$$

$$e_o\left(1 + \frac{A}{\tau s+1}\right) = \frac{A}{\tau s+1} e_i$$

$$\frac{e_o}{e_i} = \frac{A}{\tau s+1+A}$$

$$\tau s+1+A = 0$$

$$s = \frac{-1-A}{\tau}$$

Negative real root

stable

$$e_o = \frac{A}{\tau s+1}\left(e_o - e_i\right)$$

$$e_o\left(\frac{A}{\tau s+1} - 1\right) = \frac{A}{\tau s+1} e_i$$

$$\frac{e_o}{e_i} = \frac{A}{-\tau s-1+A}$$

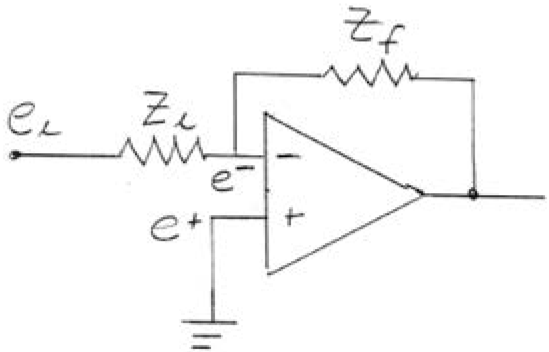
$$-\tau s-1+A = 0$$

$$s = \frac{A-1}{\tau}$$

positive real root

Unstable

5.



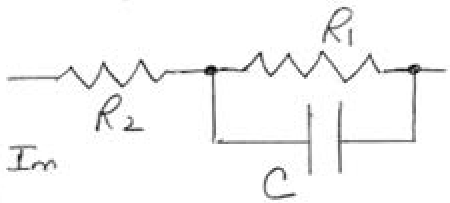
$e^+ = e^-$

$i^+ = i^- = 0$ Ideal Op-Amp

$$\frac{e_o}{e_i} = -\frac{Z_f}{Z_i}$$

(a) $Z_f \rightarrow$

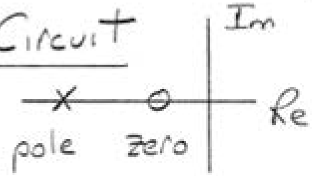
$Z_i \rightarrow$



$$Z_f = R_f$$

$$Z_i = R_2 + \frac{R_1/Cs}{R_1 + \frac{1}{Cs}}$$

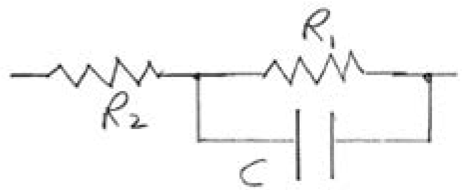
Lead Circuit



$$\frac{e_o}{e_i} = \frac{-\frac{R_f}{R_2}\left(s + \frac{1}{R_1 C}\right)}{\left(s + \frac{1}{R_1 C} + \frac{1}{R_2 C}\right)}$$

zero : $s = -\frac{1}{R_1 C}$ pole: $s = -\frac{1}{R_1 C} - \frac{1}{R_2 C}$

(b) $Z_f \rightarrow$

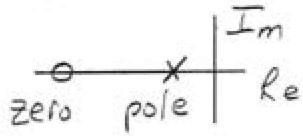


$$Z_f = R_2 + \frac{R_1/Cs}{R_1 + \frac{1}{Cs}}$$

$$Z_i = R_i$$

$Z_i \rightarrow$

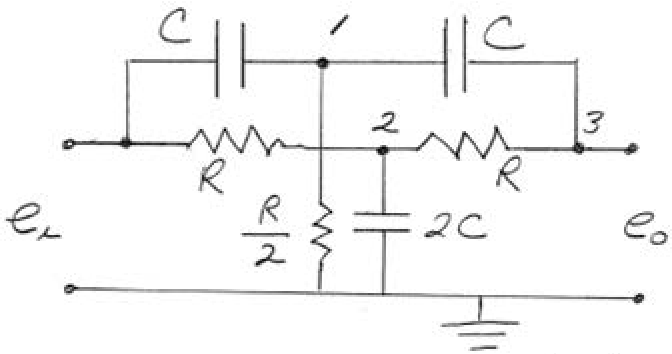
Lag Circuit



pole and zero are reversed.

$$\frac{e_o}{e_i} = \frac{-\frac{R_2}{R_i}\left(s + \frac{1}{R_1 C} + \frac{1}{R_2 C}\right)}{\left(s + \frac{1}{R_1 C}\right)}$$

(c)



Notch Circuit

KCL

Node 1: $$C\frac{d}{dt}(e_i - e_1) + \frac{0 - e_1}{R/2} + C\frac{d}{dt}(e_o - e_1) = 0$$

Node 2: $$\frac{e_i - e_2}{R} + 2C\frac{d}{dt}(0 - e_2) + \frac{e_o - e_2}{R} = 0$$

Node 3: $$C\frac{d}{dt}(e_1 - e_o) + \frac{e_2 - e_o}{R} = 0$$

Laplace Transformed Equations

$$\begin{cases} Cs(e_i - e_1) - \dfrac{2e_1}{R} + Cs(e_o - e_1) = 0 \\ \dfrac{e_i - e_2}{R} - 2Cse_2 + \dfrac{e_o - e_2}{R} = 0 \\ Cs(e_1 - e_o) + \dfrac{e_2 - e_o}{R} = 0 \end{cases}$$

Matrix Representation of Equations

$$\begin{bmatrix} -2Cs-\frac{2}{R} & 0 & Cs \\ 0 & -2Cs-\frac{2}{R} & \frac{1}{R} \\ Cs & 1/R & -Cs-\frac{1}{R} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_o \end{bmatrix} = \begin{bmatrix} -Cs \\ -\frac{1}{R} \\ 0 \end{bmatrix} e_i$$

Solve for e_o

$$\frac{e_o}{e_i} = \frac{s^2 + \frac{1}{R^2C^2}}{s^2 + \frac{4}{RC}s + \frac{1}{R^2C^2}}$$