Overhead-Crane Pneumatic-Gantry

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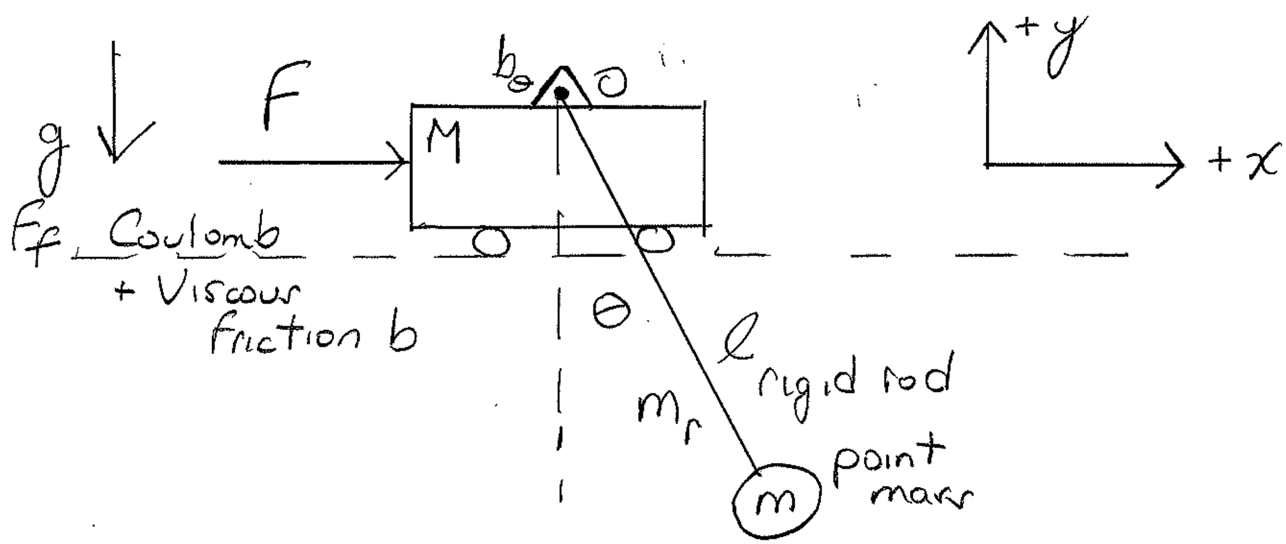
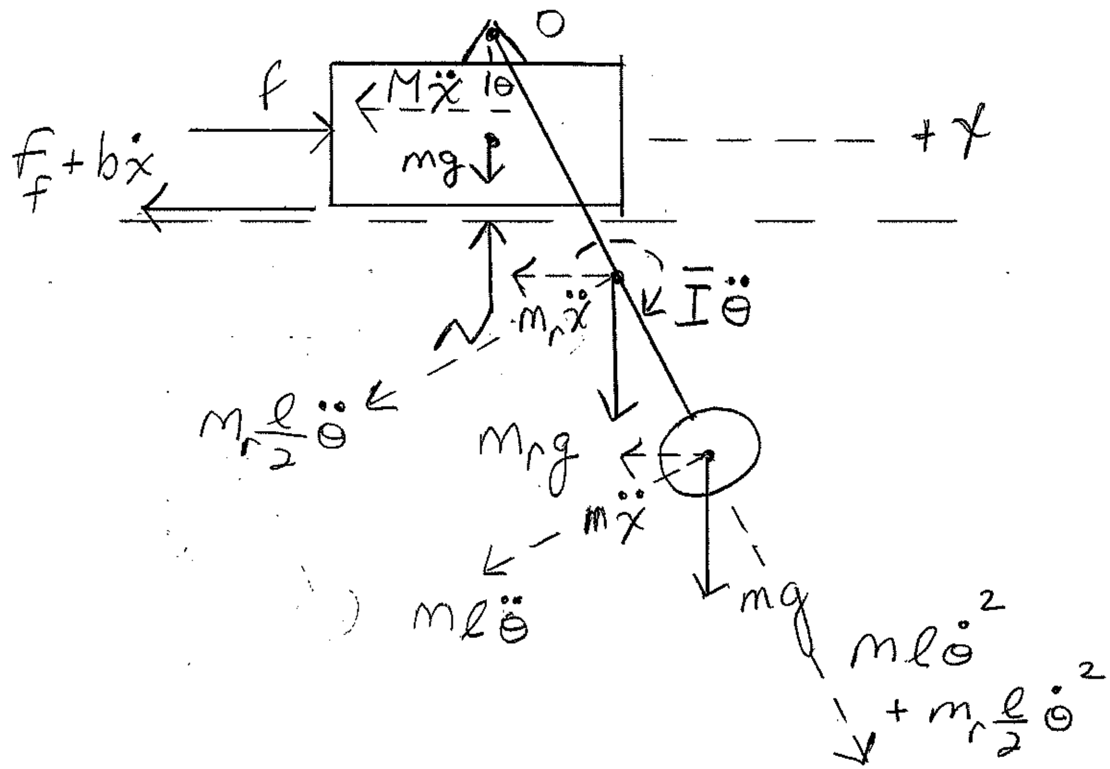


2 DOF: x θ



$\sum f_x = 0$ (cart + pendulum FBD)

$$-f_f - b\dot{x} - M\ddot{x} - m_r\ddot{x} - m\ddot{x} - m_r\frac{\ell}{2}\ddot{\theta}\cos\theta - m\ell\ddot{\theta}\cos\theta$$
$$+ m\ell\dot{\theta}^2\sin\theta + m_r\frac{\ell}{2}\dot{\theta}^2\sin\theta + F = 0$$

$$\left(M + m_r + m\right)\ddot{x} + \left(m_r\frac{\ell}{2} + m\ell\right)\cos\theta\,\ddot{\theta} - \left(m\ell + m_r\frac{\ell}{2}\right)\sin\theta\,\dot{\theta}^2$$
$$+ b\dot{x} + F_f = F$$

$\sum M_o = 0$ ⟲+ (pendulum FBD)

$$-\bar{I}\ddot{\theta} - m\ddot{x}\ell\cos\theta - m_r\ddot{x}\frac{\ell}{2}\cos\theta - mg\ell\sin\theta - m_r g\frac{\ell}{2}\sin\theta$$
$$-m\ell^2\ddot{\theta} - m_r\left(\frac{\ell}{2}\right)^2\ddot{\theta} - b_\theta\dot{\theta} = 0$$

$$\left(m\ell^2 + m_r\frac{\ell^2}{4}\right)\ddot{\theta} + \left(m\ell + m_r\frac{\ell}{2}\right)g\sin\theta + \bar{I}\ddot{\theta} + \left(m\ell + m_r\frac{\ell}{2}\right)\ddot{x}\cos\theta$$
$$+ b_\theta\dot{\theta} = 0$$

$$\bar{I} = \frac{1}{12}m_r\ell^2$$

Summary

$$(M+m_r+m)\ddot{x} + b\dot{x} + F_f + \left(\frac{m_r \ell}{2} + m\ell\right)\cos\theta\,\ddot{\theta}$$
$$- \left(m\ell + \frac{m_r \ell}{2}\right)\sin\dot{\theta}^2 = F \qquad [1]$$

$$\left(m\ell^2 + m_r\frac{\ell^2}{4}\right)\ddot{\theta} + \left(\frac{1}{12}m_r\ell^2\right)\ddot{\theta} + \left(m\ell + \frac{m_r\ell}{2}\right)g\sin\theta$$
$$+\left(m\ell + \frac{m_r\ell}{2}\right)\ddot{x}\cos\theta + b_\theta\dot{\theta} = 0 \qquad [2]$$

Assume : $\sin\theta \approx \theta \quad \cos\theta \approx 1 \quad \dot{\theta}^2 \approx 0$

$$(M+m_r+m)\ddot{x} + b\dot{x} + F_f + \left(\frac{m_r}{2}+m\right)\ell\ddot{\theta} = F \qquad [3]$$

$$\left(m\ell^2 + \frac{m_r\ell^2}{4} + \frac{1}{12}m_r\ell^2\right)\ddot{\theta} + \left(m+\frac{m_r}{2}\right)\ell g\theta + \left(m+\frac{m_r}{2}\right)\ell\ddot{x} = 0$$

$$\left(m+\frac{m_r}{3}\right)\ell^2\ddot{\theta} + \left(m+\frac{m_r}{2}\right)\ell g\theta + \left(m+\frac{m_r}{2}\right)\ell\ddot{x} + b_\theta\dot{\theta} = 0$$

$$\left(m+\frac{m_r}{3}\right)\ell\ddot{\theta} + \left(m+\frac{m_r}{2}\right)g\theta + \left(m+\frac{m_r}{2}\right)\ddot{x} + \frac{b_\theta}{\ell}\dot{\theta} = 0 \qquad [4]$$

Linear Equations (F_f = Coulomb friction - nonlinear)

$$(M + m_r + m)\ddot{x} + b\dot{x} + \left(\frac{m_r}{2} + m\right)\ell\ddot{\theta} = F$$

$$\left(m + \frac{m_r}{3}\right)\ell\ddot{\theta} + \left(m + \frac{m_r}{2}\right)g\theta + \left(m + \frac{m_r}{2}\right)\ddot{x} + \frac{b_\theta}{\ell}\dot{\theta} = 0$$

Define

$$\left(m + \frac{m_r}{2}\right) \equiv c_2 \qquad \left(m + \frac{m_r}{3}\right) \equiv c_3$$

$$(M + m_r + m) \equiv c_1 \qquad \frac{b_\theta}{\ell} \equiv d$$

$$c_1\ddot{x} + b\dot{x} + c_2\ell\ddot{\theta} = F \qquad [5]$$

$$c_3\ell\ddot{\theta} + c_2 g\theta + c_2\ddot{x} + d\dot{\theta} = 0 \qquad [6]$$

$$C_1\ddot{x} + b\dot{x} + C_2 l\ddot{\theta} = F \longrightarrow \ddot{\theta} = \frac{1}{C_2 l}\left[F - C_1\ddot{x} - b\dot{x}\right] \quad 4a$$

$$C_3 l\ddot{\theta} + C_2 g\theta + C_2\ddot{x} + d\dot{\theta} = 0 \longrightarrow \ddot{x} = \frac{1}{C_2}\left[-C_3 l\ddot{\theta} - C_2 g\theta - d\dot{\theta}\right]$$

$$\left\{\begin{array}{l} C_1\left[\frac{1}{C_2}\left(-C_3 l\ddot{\theta} - C_2 g\theta - d\dot{\theta}\right)\right] + b\dot{x} + C_2 l\ddot{\theta} = F \\ -\frac{C_1}{C_2}C_3 l\ddot{\theta} - C_1 g\theta - \frac{C_1}{C_2}d\dot{\theta} + b\dot{x} + C_2 l\ddot{\theta} = F \\ \left[C_2 l - \frac{C_1 C_3}{C_2}\right]l\ddot{\theta} - \frac{C_1}{C_2}d\dot{\theta} - C_1 g\theta + b\dot{x} = F \quad [5a] \end{array}\right.$$

$$C_3 l\left[\frac{1}{C_2 l}\left(F - C_1\ddot{x} - b\dot{x}\right)\right] + C_2 g\theta + C_2\ddot{x} + d\dot{\theta} = 0$$

$$\frac{C_3}{C_2}F - \frac{C_1 C_3}{C_2}\ddot{x} - \frac{C_3}{C_2}b\dot{x} + C_2 g\theta + C_2\ddot{x} + d\dot{\theta} = 0$$

$$\left[C_2 - \frac{C_1 C_3}{C_2}\right]\ddot{x} - \frac{C_3}{C_2}b\dot{x} + C_2 g\theta + d\dot{\theta} = -\frac{C_3}{C_2}F \quad [6a]$$

$$\begin{bmatrix} (C_1 s^2 + bs) & C_2 \ell s^2 \\ C_2 s^2 & (C_3 \ell s^2 + ds + C_2 g) \end{bmatrix} \begin{bmatrix} X \\ \Theta \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

$$\begin{aligned} Den &= (C_1 s^2 + bs)(C_3 \ell s^2 + ds + C_2 g) - (C_2 \ell s^2)(C_2 s^2) \\ &= C_1 C_3 \ell s^4 + C_1 d s^3 + C_1 C_2 g s^2 + b C_2 g s \\ &\quad + C_3 b \ell s^3 + b d s^2 \\ &= (C_1 C_3 \ell) s^4 + (C_1 d + C_3 b \ell) s^3 + (C_1 C_2 g + bd) s^2 + (b C_2 g) s \end{aligned}$$

$$Num\_X = (F)(C_3 \ell s^2 + ds + C_2 g)$$

$$Num\_\Theta = (F)(C_2 s^2)$$

$$G_1(s) = \frac{\Theta}{X}(s) = \frac{C_2 s^2}{C_3 l s^2 + d s + C_2 g}$$

$$G_2(s) = \frac{X}{F}(s) = \frac{C_3 l s^2 + d s + C_2 g}{(C_1 C_3 l)s^4 + (C_1 d + C_3 b l)s^3 + (C_1 C_2 g + bd)s^2 + (bC_2 g)s}$$

$$G_3(s) = \frac{\Theta}{F}(s) = \frac{C_2 s^2}{(C_1 C_3 l)s^4 + (C_1 d + C_3 b l)s^3 + (C_1 C_2 g + bd)s^2 + (bC_2 g)s}$$