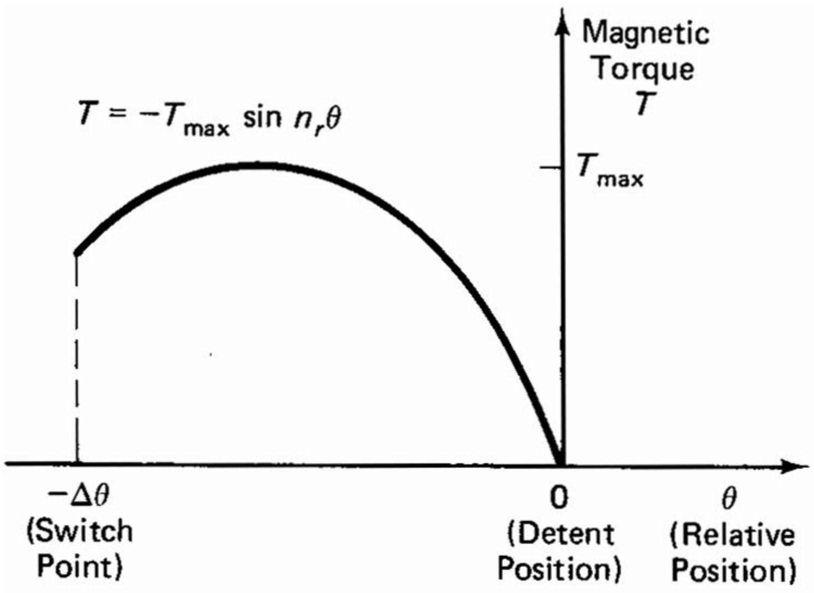
Step Motor Modeling

- Stepper Motor Models
  - Under steady operation at low speeds, we usually do not need to differentiate between VR motors and PM motors (a hybrid motor is a special type of PM motor).
  - But under transient conditions, the torque characteristics of the two types of motors can differ substantially.
  - The torque in a PM motor varies somewhat linearly with magnitude of the phase current.
  - The torque in a VR motor varies nearly quadratically with the phase current.
  - Under steady-state operation of a stepper motor at low speeds, the magnetic torque can be approximated by a sinusoidal function.

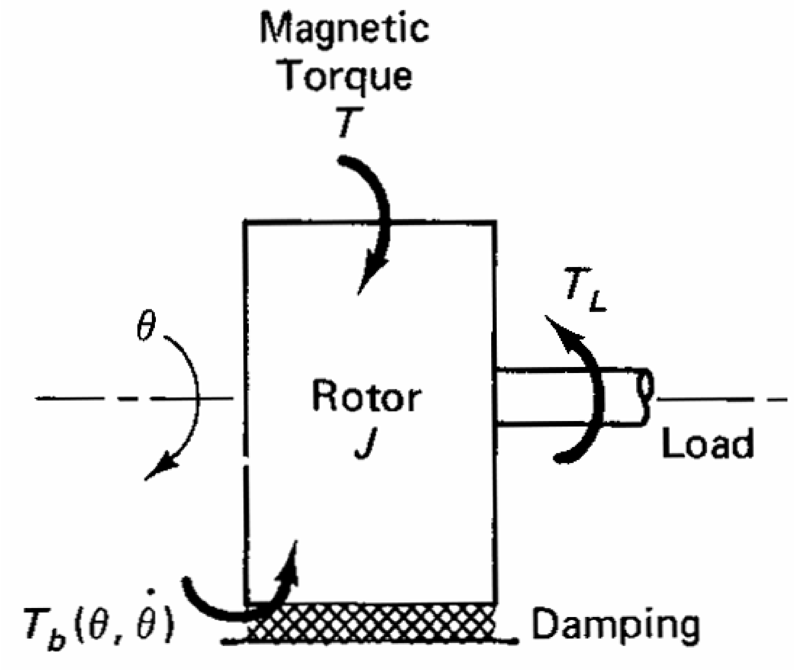
$$T = -T_{max} \sin\left(n_r \theta\right)$$

– Note that θ is the angular position of the rotor measured from the detent position of the presently excited phase. It gives the relative position of the rotor during each step. The absolute position is obtained by adding θ to the absolute rotor angle at the approaching detent position.



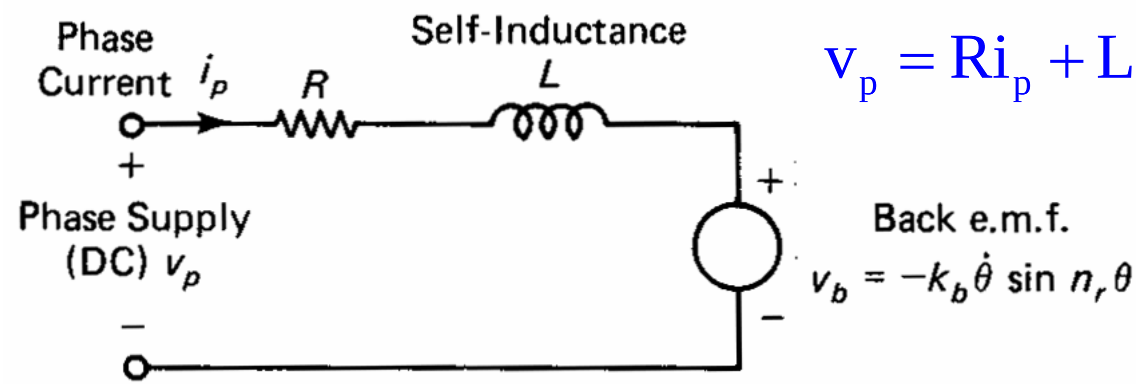
– The motor mechanical equation of motion is given by:

$$T - T_L - T_b\left(\theta, \dot{\theta}\right) = J\ddot{\theta}$$



- Under high-speed and transient operation of a stepper motor, many of the quantities that were assumed constant will vary with time as well as rotor position.
- In particular, for a given supply voltage to a phase winding, the associated phase current will not be constant.
- Furthermore, inductance L in the phase circuit will vary with the rotor position.
- Also a back emf will be induced in the phase circuit because of the magnetic flux changes resulting from the speed of rotation of the rotor.
- An improved dynamic model would be needed to represent the behavior of a stepper motor under high-speed and transient conditions.

– An approximate equivalent circuit for one phase of a stepper motor (neglecting mutual inductance) is shown below.



$$v_p = Ri_p + L\frac{di_p}{dt} + v_b$$

– Since θ is negative in a conventional step (from θ = -Δθ to θ = 0), we note that v_b is positive for positive angular velocity.

– Self-inductance also varies with rotor position θ and is periodic with rotor tooth pitch: $L = L_0 + L_a \cos(n_r \theta)$

– The model so far is valid for both types of stepper motors, PM and VR. But the torque equation will depend on the type of stepper motor.

– Torque Equation for PM Motors

- In a permanent-magnet stepper motor, the magnetic flux is generated by both the phase current i_p and the magnetized rotor. The flux from the magnetic rotor is constant, but its linkage with the phase windings will be modulated by the rotor position θ.
- i_p is the phase current and k_m is the torque constant for the PM motor.

$$T = -k_m i_p \sin\left(n_r \theta\right)$$

– Torque Equation for VR Motors

- In a variable-reluctance stepper motor, the rotor is not magnetized; hence, there is no magnetic flux generation from the rotor. The flux generated by the phase current i_p is linked with the phase windings. The flux linkage is modulated by the motion of the VR motor, however.
- k_r is the torque constant of the VR motor
- Note that torque depends on the phase current i_p in a quadratic manner in the VR stepper motor.

$$T = -k_r i_p^2 \sin\left(n_r \theta\right)$$

– Summary

- To compute the torque T at a given rotor position, we have to solve the following differential equation for known values of the rotor position θ and the rotor speed and for a given (constant) phase supply voltage v_p.

$$v_p = Ri_p + L\frac{di_p}{dt} + v_b$$

$$v_b = -k_b\dot{\theta}\sin(n_r\theta)$$

$$L = L_0 + L_a\cos(n_r\theta)$$

- The model parameters R, L_0, L_a, and k_b are assumed to be known (experimentally or from the manufacturer's data sheet).

– The torque is computed using:

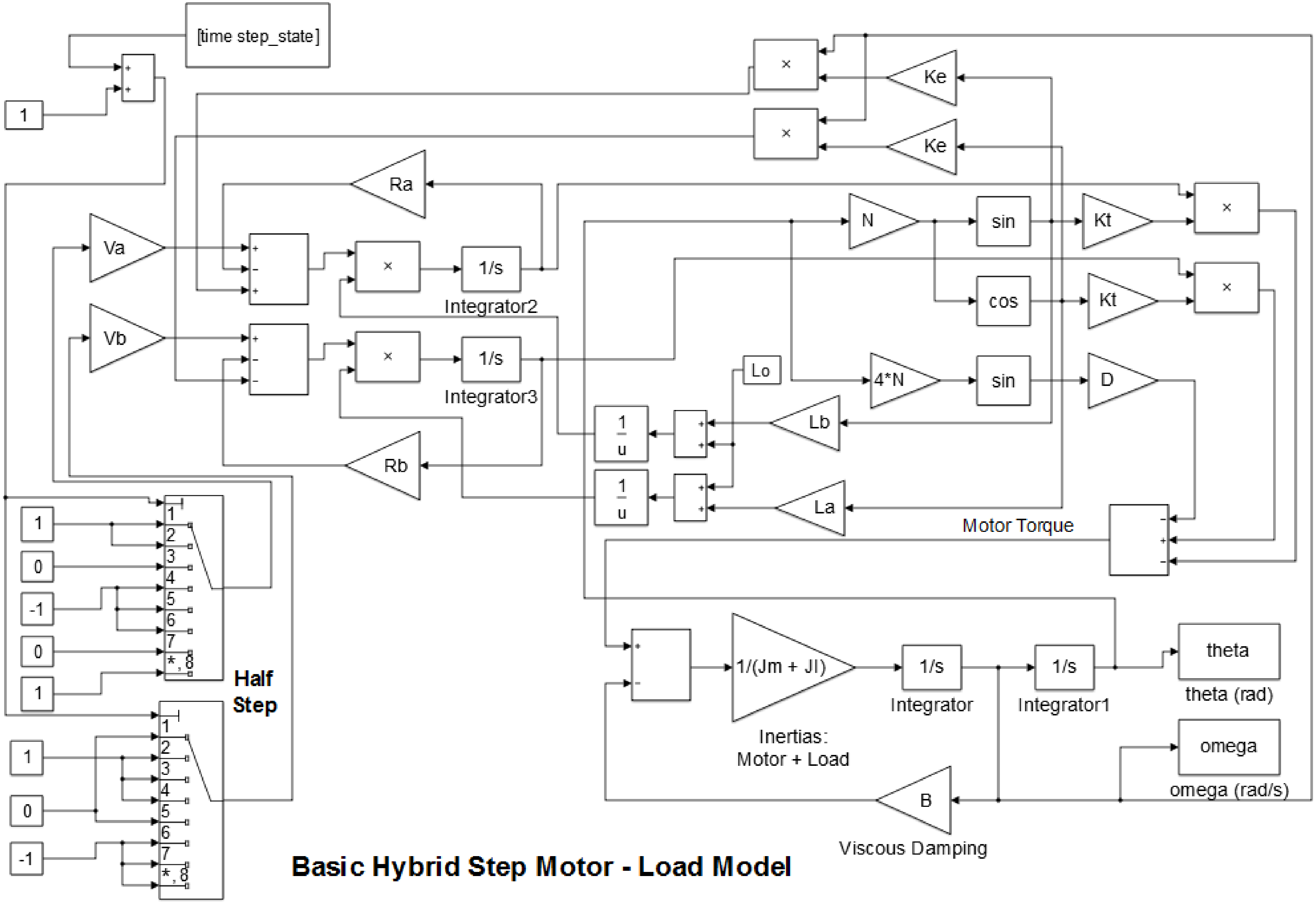
$$T = -k_m i_p \sin(n_r \theta) \qquad \text{PM Stepper Motor}$$

$$T = -k_r i_p^2 \sin(n_r \theta) \qquad \text{VR Stepper Motor}$$

– Again, the torque constant (k_m or k_r) is assumed to be known.

– The simulation of the model then can be completed by using this torque in the mechanical dynamic equation to determine rotor position and rotor speed:

$$T - T_L - T_b\left(\theta, \dot{\theta}\right) = J\ddot{\theta}$$



Basic Hybrid Step Motor - Load Model