System Inputs, Physical Modeling, and Time & Frequency Domains

There are three topics that require more discussion at this point of our study. They are: Classification of System Inputs, Physical Modeling, and Time Domain vs. Frequency Domain. Let's develop each of these further.

## Classification of System Inputs

Questions:
What is the input to the actual physical system that we are measuring the response to?
What is the input to the mathematical model that we are predicting the response to?
Of course, these two inputs must be the same if we are to compare the measured response to the predicted response.
Are there standard inputs used by engineers in the investigation process?
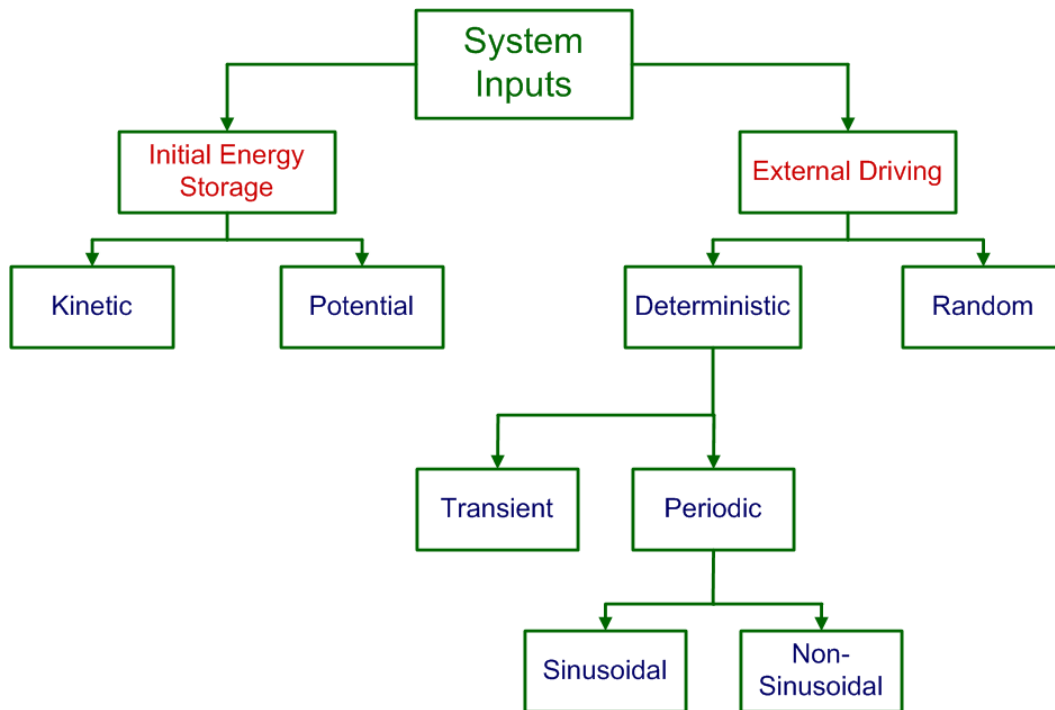If so, what are they? Why are they effective? Why not use the actual real-world inputs?
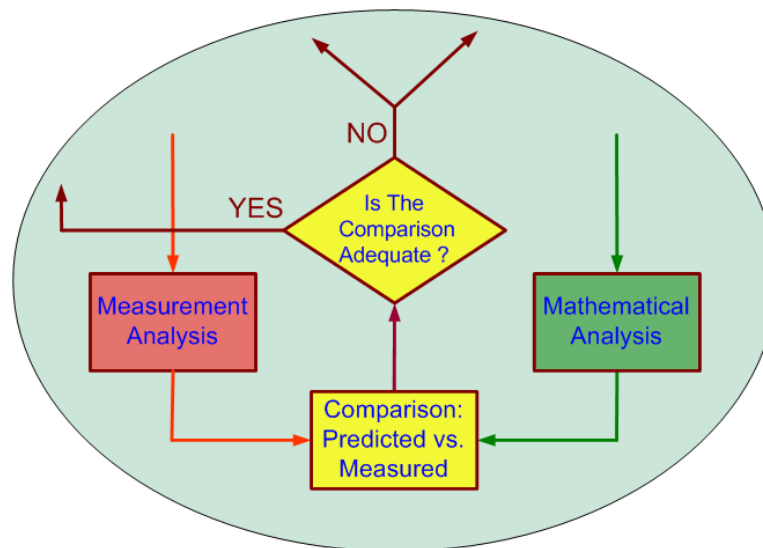
The model of our physical system under investigation, both physical and mathematical, must be validated, i.e., its soundness must be confirmed, if it is to be of any use. How does an engineer validate a model of a physical system? In order to validate the physical and mathematical model of the physical system under investigation, the engineer must first cause the system to respond. This is done by introducing to both the system mathematical model and to the actual physical system an input. An input is simply some action which will cause the system to respond. The same input is introduced to both the mathematical model and to the physical system. The predicted response is obtained by solving the mathematical model, i.e., the equations describing the behavior of the physical model, with the designated input. The actual response is obtained by introducing into the actual physical system the designated input and measuring the response using instruments like multimeters, oscilloscopes, or dynamic signal analyzers.

What types of system inputs are there? The diagram shows one possible classification of system inputs. The two main classifications of system inputs are initial energy storage and

external driving. Let's use as an example the common, simple, spring-mass-damper mechanical system to illustrate the difference.

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                          ┌──────────────┐
                          │   System     │
                          │   Inputs     │
                          └──────────────┘
              ┌───────────────────┴────────────────────┐
     ┌──────────────────┐                     ┌──────────────────┐
     │  Initial Energy  │                     │ External Driving │
     │     Storage      │                     └──────────────────┘
     └──────────────────┘                 ┌───────────┴───────────┐
      ┌───────┴───────┐         ┌──────────────────┐      ┌──────────────┐
┌──────────┐  ┌──────────────┐  │  Deterministic   │      │    Random    │
│ Kinetic  │  │  Potential   │  └──────────────────┘      └──────────────┘
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                              ┌──────────────┐  ┌──────────────┐
                              │  Transient   │  │  Periodic    │
                              └──────────────┘  └──────────────┘
                                              ┌─────┴──────┐
                                      ┌──────────────┐  ┌──────────────┐
                                      │  Sinusoidal  │  │    Non-      │
                                      └──────────────┘  │  Sinusoidal  │
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Without an input, the mass remains stationary in an equilibrium state with the weight of the mass balanced by the spring force. How can we put the mass into motion? One way is to simply pull the mass down and release it. The mass will then oscillate up and down and eventually come to rest. When we pull the mass down we increase the stretch of the spring, storing energy in the spring as potential energy. When we release the mass, this stored-up potential energy is then given up causing the mass to move. Another way we could put the mass into motion would be to give it an initial velocity. One could hit the mass, i.e., give it an impulsive force, and this would result in an initial velocity of the mass and resulting oscillation. In this case kinetic energy is given to the mass initially. Both of these cases are examples of a system input classified as initial energy storage. The first being an example of potential energy storage and the second being an example of kinetic energy storage. Once we release the system, no external driving action is needed to keep the system in motion. An engineer might measure the position, velocity, or acceleration of the mass as it moves and these would be called the outputs of the system. So we see initial energy storage refers to a situation where the engineer puts a system, initially in an equilibrium state, into a different state and then releases the system. The system then responds free from any external interference. We could have used as an example an electrical, electromagnetic, thermal, or fluid system.

The other major classification of input is external driving. In this case, a physical quantity from the system's environment, i.e., from outside the system boundary, is applied to the system and causes it to respond. We often choose to study the system response to an assumed ideal source, which is unaffected by the system to which it is coupled, with the view that practical situations will closely correspond to this idealized model. External inputs can be

broadly classified as deterministic or random, recognizing that there is always some element of randomness and unpredictability in all real-world inputs. Deterministic input models are those whose complete time history is explicitly given, as by mathematical formula or a table of numerical values. This can be further divided into two categories. A transient input model is one having any desired shape, but existing only for a certain time interval, being constant before the beginning of the interval and after its end. An example of this type of input for the spring-mass system would be a constant force applied at some instant and then removed at some later instant. The constant force could be applied just by adding an additional mass at some initial time and then removing it at a later time. The response of the system would be observed both before, during, and after the application of the force. The second type of deterministic input is a periodic input model. This input type repeats a certain wave form over and over, ideally forever, and is further classified as either sinusoidal or non-sinusoidal. For the spring-mass system, an actuator (e.g., a motor with a rack-and-pinion gear) could apply a sinusoidal-varying or non-sinusoidal-varying force to the mass.

Random input models are the most realistic input models and have time histories which cannot be predicted before the input actually occurs, although statistical properties of the input can be specified. When working with random inputs, there is never any hope of predicting a specific time history before it occurs, but statistical predictions can be made that have practical usefulness.

Engineers typically use two inputs to evaluate dynamic systems: a step input and a sinusoidal input. By a step input of any variable, we will always mean a situation where the system is at rest at time t = 0 and we instantly change the input quantity, from wherever it was just before t = 0, by a given amount, either positive or negative, and then keep the input constant at this new value forever. This leads to a transient response called the step response of the system. When the input to the system is a sine wave, the steady-state response of the system, after all the transients have died away, is called the frequency response of the system. These two input types lead to the two views of dynamic system response: time response and frequency response.

Why only use these two types of inputs to evaluate a dynamic system? The practical difficulty is that precise mathematical functions for actual real-world inputs will not generally be known in practice. Therefore the random nature of many practical inputs makes difficult the development of performance criteria based on the actual inputs experienced by real system. It is thus much more common to base performance evaluation on system response to simple "standard" inputs – step input and sine wave input. This approach has been successful for several reasons:
- Experience with the actual performance of various classes of systems has established a good correlation between the response of systems to these standard inputs and the capability of the systems to accomplish their required tasks.
- Design is much concerned with comparison of competitive systems. This comparison can often be made nearly as well in terms of standard inputs as for real inputs.
- Simplicity of form of standard inputs facilitates mathematical analysis and experimental verifications.
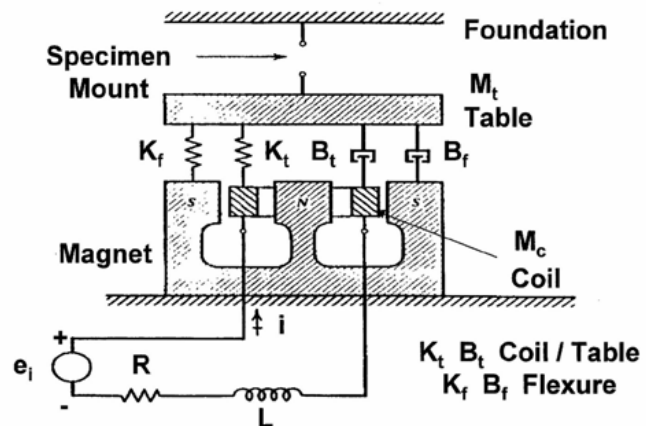
Physical Modeling

The difference between a physical system and a physical model is dramatically illustrated by the electrodynamic vibration exciter. The structure of this device is similar to a common loudspeaker, shown below. It delivers a force proportional to the current applied to its coil and is also very similar in operation to the electro-pneumatic transducer. On the left below is one that fits in the palm of your hand, and on the right below is one that weighs 600 lbs and is capable of delivering a force of 6000 lbs.
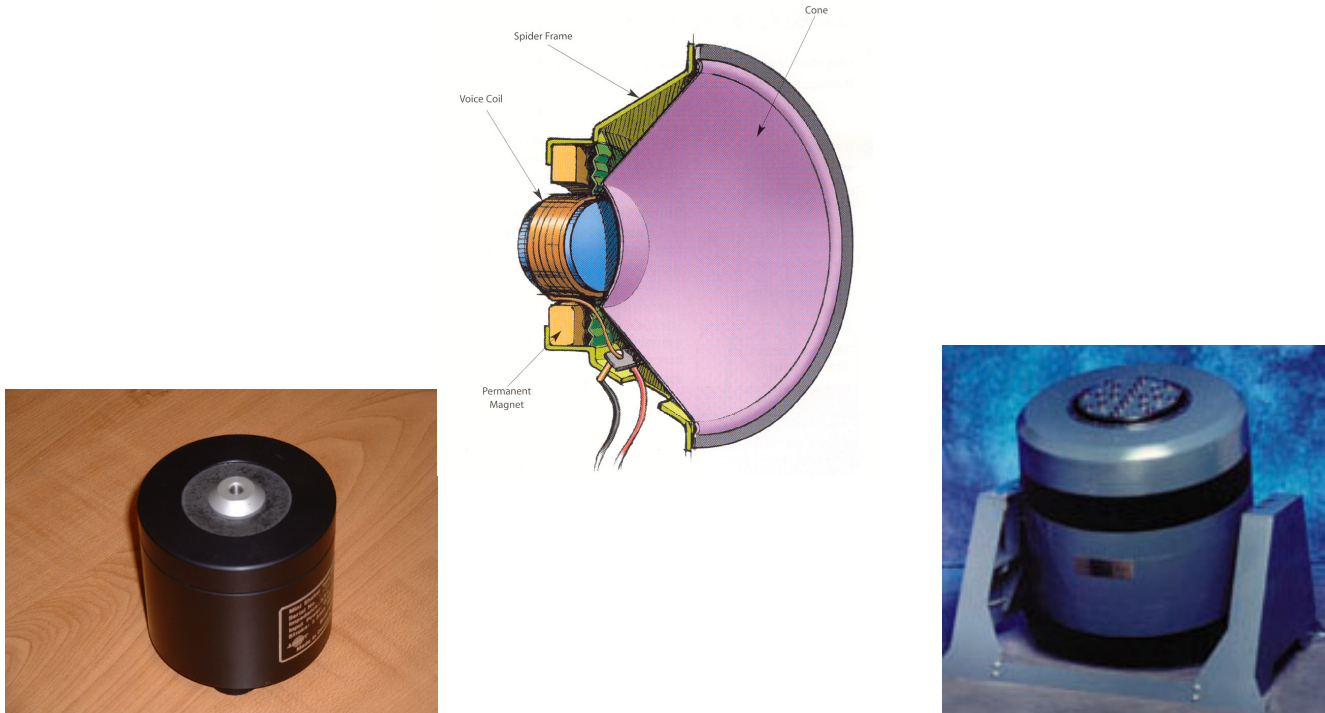
Shown on the left below is a schematic of an electrodynamic vibration exciter. This "moving coil" type of device converts an electrical command signal into a mechanical force and/or motion and is very common, e.g., vibration shakers, loudspeakers, linear motors for positioning heads on computer disk memories, and optical mirror scanners. In all these cases, a current-carrying coil is located in a steady magnetic field provided by permanent magnets in small devices and electrically-excited wound coils in large ones. Two electromechanical effects are observed in such configurations: Generator Effect – motion of the coil through the magnetic field causes a voltage proportional to velocity to be induced into the coil, and Motor Effect – passage of current through the coil causes it to experience a magnetic force proportional to the current.
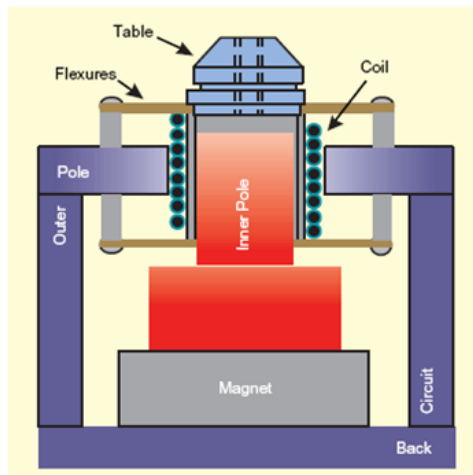
Physical System · Physical Model

A physical model of this device is shown in the diagram at the right above. While the details of the modeling process will occupy much of our attention, the key concept is that this device has distributed throughout it several characteristics - mass, compliance, energy dissipation, electrical resistance, and electrical inductance. We capture these distributed characteristics in our physical model through the use of idealized elements that are considered pure and ideal. A pure element has only the characteristic for which it is named, e.g., a pure spring only has compliance, while a real spring has compliance, mass, and energy dissipation when cycled. An ideal element behaves in a linear manner, i.e., the output is linearly proportional to the input. Most real-world devices do not behave in a linear fashion.

Flexure K_f is an intentional soft spring (stiff, however, in the radial direction) that serves to guide the axial motion of the coil and table. Flexure damping B_f is usually intentional, fairly strong, and obtained by laminated construction of the flexure spring, using layers of metal, elastomer, plastic, and so on. The coupling of the coil to the shaker table would ideally be rigid so that the magnetic force is transmitted undistorted to the mechanical load. Thus, K_t (generally large) and B_t (quite small) represent parasitic effects rather than intentional spring and damper elements. R and L are the total circuit resistance and inductance, including contributions from both the shaker coil and the amplifier output circuit.

As an added example of the difference between a physical system and a physical model, consider the real spring shown below on the left. It has mass, compliance, and energy dissipation characteristics distributed throughout it. A physical model of this real spring, using pure and ideal spring, mass, and damper elements to capture these essential characteristics, is shown below on the right. The two challenges observed here are first, the structure or connection of the pure and ideal elements, and second, the parameter values of these pure and ideal elements. If the modeling is done properly and the parameter values are identified correctly, then the mathematical model will predict the same response as measured on the actual physical system, for the range of operation the model is intended for.
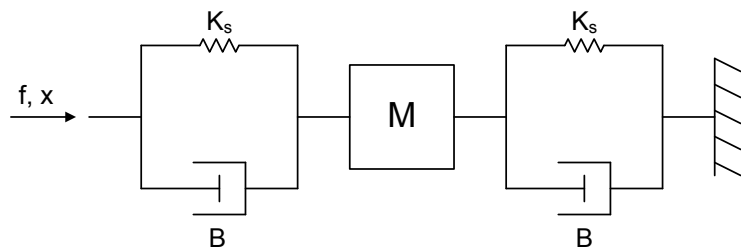
Time Domain vs. Frequency Domain

Time domain and frequency domain are two ways of looking at the same dynamic system. They are interchangeable, i.e., no information is lost in changing from one domain to another. They are complementary points of view that lead to a complete, clear understanding of the behavior of a dynamic engineering system.

The time domain is a record of the response of a dynamic system, as indicated by some measured parameter, as a function of time. This is the traditional way of observing the output of a dynamic system. An example of time response is the displacement of the mass of the spring-mass-damper system versus time in response to the sudden placement of an additional mass (here 50% of the attached mass) on the attached mass. The resulting response is the step response of the system due to the sudden application of a constant force to the attached mass equal to the weight of the additional mass. Typically when we investigate the performance of a dynamic system we use as the input to the system a step input.
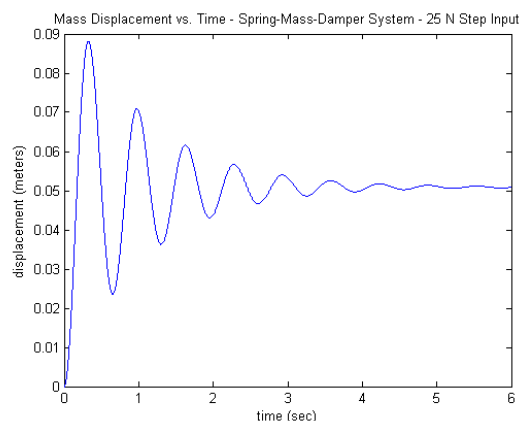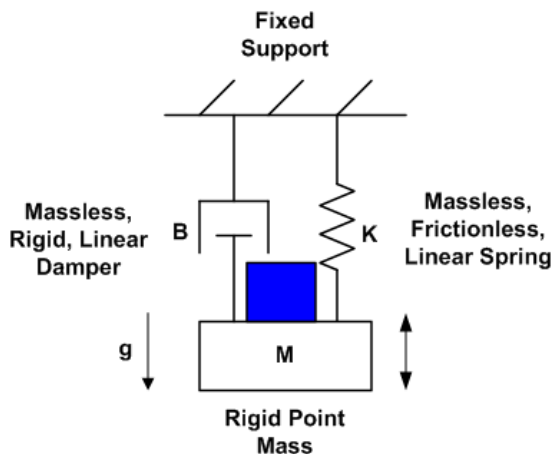

Over one hundred years ago, Jean Baptiste Fourier showed that any waveform that exists in the real world can be generated by adding up sine waves. By picking the amplitudes, frequencies, and phases of these sine waves, one can generate a waveform identical to the desired signal. While the situation presented below is contrived, it does illustrate the idea. On

the left is a "real-world" signal and on the right are three signals, the sum of which is the same as the "real-world" signal.

Real-World Signal


Real-World Signal = Sum of Three Signals
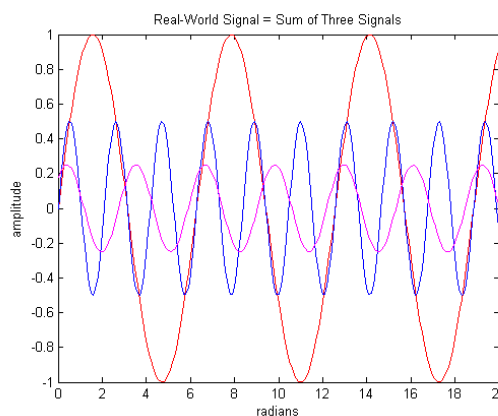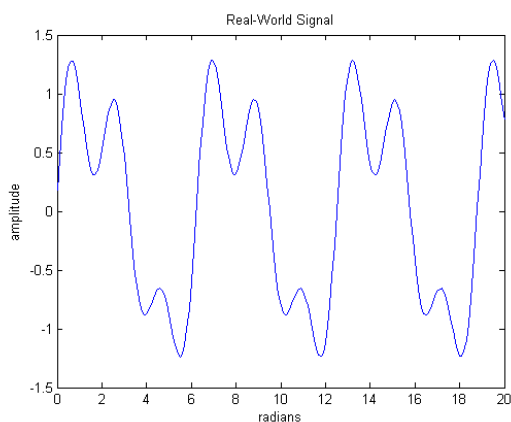
A more convincing example is to observe that a square wave can be represented by a series of sine waves of different amplitudes, frequencies, and phase angles. In the diagram at the right, a square wave has been approximated with only two sine waves. As more sine waves are added to the series, the approximation becomes better and better.


Square Wave Appoximated by Two Sine Waves
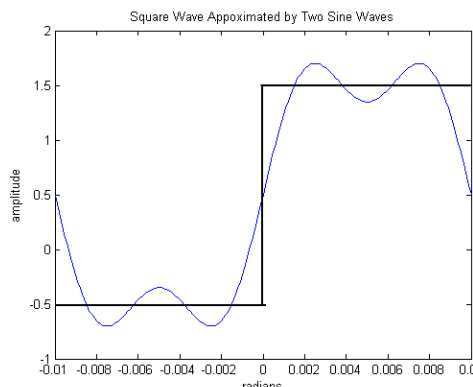
Any real-world signal can be broken down into a sum of sine waves and this combination of sine waves is unique. Any real-world signal can be represented by only one combination of sine waves.

In the diagram at the right, a waveform is represented as the sum of two sine waves. Below, in figure (a), is a three-dimensional graph of this addition of sine waves. The three axes are time, amplitude, and frequency. The time and amplitude axes are familiar from the time domain. The third axis,
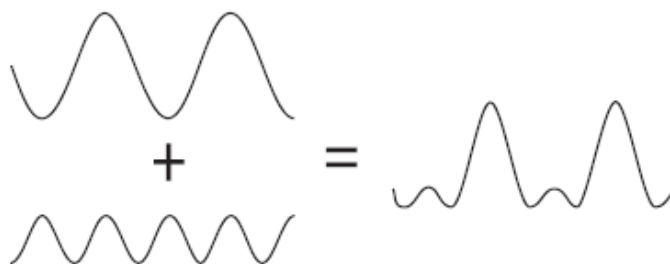
frequency, allows us to visually separate the sine waves that add to give us the complex waveform. If we view this three-dimensional graph along the frequency axis, we get the view shown in figure (b). This is the time-domain view of the sine waves. Adding them together at each instant of time gives the original waveform. Now view the three-dimensional graph along

the time axis, as in figure (c). Here we have axes of amplitude versus frequency. This is what is called the frequency domain. Every sine wave we separated from the input appears as a vertical line. Its height represents its amplitude and its position represents its frequency. The phase of each sine wave is not shown here. We know each line represents a sine wave and so we have uniquely characterized our input signal in the frequency domain. This frequency domain representation of our signal is called the spectrum of the signal. Each sine wave line of the spectrum is called a component of the total signal.

Time & Frequency Domains
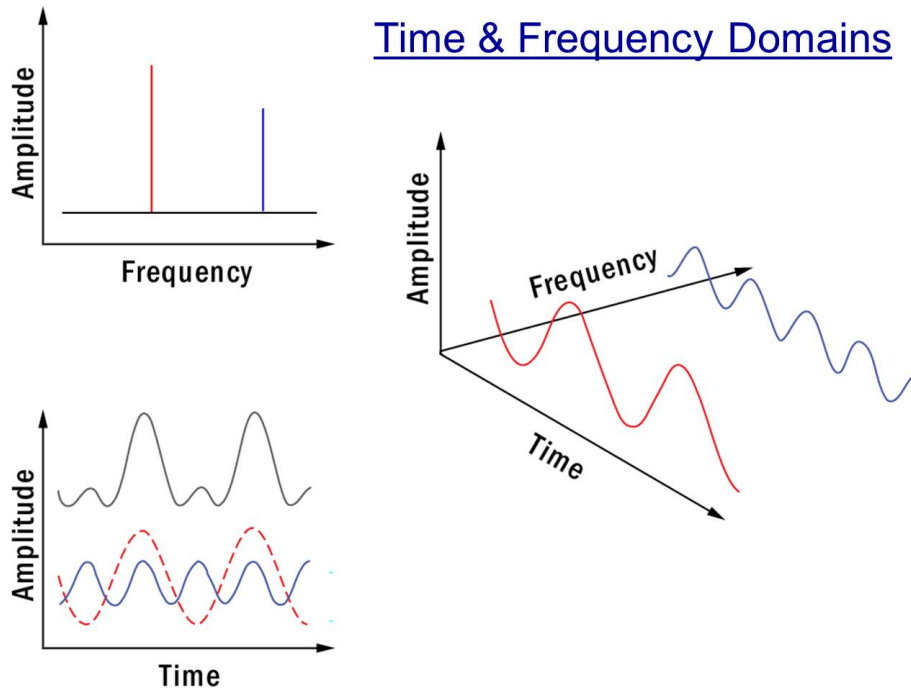
It is most important to understand that we have neither gained nor lost information; we are just representing it differently. You can now see why a sine wave is the second important signal, the step input being the other, used to excite a dynamic system. Since any real-world signal can be represented by the sum of sine waves, if we can predict the response of a system to a sine wave input of varying frequency, amplitude, and phase angle, then we can predict the response of the system to any real-world signal once we know the frequency spectrum of that real-world signal.